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Deconstruction of the Propositional Logic in Three-stage Progressive Geometry Comprehensive Test Items

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Abstract

Geometry comprehensive examination items in Chinese secondary school mathematics education are used to assess students and to cultivate their higher-order thinking abilities. Among all the forms of study that have been performed in practice, the three-stage progressive geometry examination item has become a typical design; that is to say, it increases the cognitive load by having three consecutive sub-questions with increasing difficulty. In accordance with the revised taxonomy of educational objectives and the Van Hiele model of geometric thinking, this paper will systematically deconstruct the propositional logic contained in such items to explore how designers organise condition setting, constraint imposition and question sequence. By analysing the structure of typical objects, the three stages in logic can be divided into: the first is fundamental recognition and recall; The second is relational application and deduction; and the third is all-round synthesis with open-ended exploration. Propounded dependencies among the sub-questions will be analyzed in terms of the structure of information sharing and the cumulative scaffolding effect. The results can offer some suggestions for mathematics teachers in designing test questions and organising teaching.

Keywords

Three-stage progressive structure; geometry examination items; propositional logic; cognitive hierarchy; mathematics education

1. Introduction

Geometry occupies a core position in China's compulsory secondary mathematics curriculum. As stipulated in the Mathematics Curriculum Standards for Compulsory Education (2022 Edition) issued by the Ministry of Education of the People's Republic of China [1], geometric intuition, spatial imagination, and deductive reasoning constitute core mathematical competencies all secondary school graduates are required to master. Provincial and national standardized examinations routinely feature high-difficulty geometry comprehensive problems, which require multi-step logical reasoning and integrate multiple branches of geometric knowledge within a single figure.

Among all mainstream geometry assessment formats, three-stage progressive items are most widely adopted. This design presents one complete geometric diagram and three connected subquestions of increasing difficulty, each built upon the premises and derived conclusions of the preceding part. Despite its widespread application, existing research lacks in-depth theoretical exploration of its propositional logic; most relevant studies merely analyze students' answering performance rather than unpacking the intrinsic logical architecture that underpins item construction.

Against this backdrop, this paper addresses three core research questions: What distinct propositional structures

define each tier of a three-stage progressive geometry item? How do logical interconnections across tiers shape an item's internal consistency and discrimination power? What standardized design principles can guide developers to construct logically rigorous, pedagogically effective test items? Section 2 establishes the theoretical foundation anchored in Bloom's revised taxonomy [2] and the Van Hiele geometric thinking model [3]. Section 3 dissects the cognitive structure of the three progressive tiers. Section 4 elaborates on the core propositional logic of item development. Section 5 summarizes evidence-based item design principles. Section 6 concludes the paper and proposes future research directions.

2. Theoretical Framework

2.1 Bloom's Taxonomy and Van Hiele's Geometric Thinking Levels

The Design of exam questions that scaffold cognitive demand must be based on principled theories of cognitive hierarchy. Bloom and his colleagues [4] proposed that the six levels of educational objectives include knowledge, comprehension, application, analysis, synthesis and evaluation; this system has been used around the world to construct tests for a long time. Anderson and Krathwohl [5] have revised these levels to be cognitive processes, such as remembering, understanding, applying, analysing, evaluating and creating, which are organised in different types of knowledge and provide a more dynamic vocabulary for describing what learners need to do at various stages in an examination item.

Van Hiele's five stages of geometric comprehension are proposed in the area of geometry: The first is to recognise shapes as whole figures; the second is to observe attributes through observation; the third is to make informal inferences and relational reasoning about attributes; the fourth is to conduct formal proofs in an axiomatic system; and the fifth is to apply rigour to several axiomatic systems [6]. A good three-stage geometry item will raise the level of thinking required by the solver and move them from recognition and direct application to relational inference and open-ended argumentation. Taken together, these frameworks indicate that in the Design of items, not only should there be problems of various difficulties, but also planned cognitive shifts need to be arranged reasonably according to logical reasons.

2.2 The Concept of Three-Stage Progression

Progression in the Design of Examinations is based on the view that mathematical competence is not monolithic. Schoenfeld believed that mathematical problem-solving includes more than just procedure; it also involves planning strategies, monitoring one's own learning in mathematics, etc., and using various knowledge flexibly. The three levels of progressive item operationalisation can provide a good study environment for students to show their all-round ability in the same geometric context.

The three stages should be distinguished from the three problems that happen to have the same shape. The main characteristic of a progressive structure is propositional dependency; that is, the conditions and results of an earlier part serve as the given information for the following part. Dependence can take the form of a direct transfer of a computed value, the reuse of a proved geometric relation, or the extension of a constructed auxiliary figure; in all these cases, the latter are logically subordinate to the former. The propositional dependency of the three-stage form makes it a different type of exercise from all others that only share a single figure, and thus it can assess both the depth of geometric knowledge and one's ability to use that knowledge flexibly in a developing logical system.

3. Structural Anatomy of Three-Stage Progressive Geometry Items

3.1 Stage One: Foundational Recognition and Recall

Stage One aligns with the lowest tiers of Bloom's revised taxonomy (remembering and understanding) and corresponds to Van Hiele Levels 1 and 2. It fulfills two core functions: establishing a unified geometric context that underpins the subsequent two stages, and enabling students to accurately identify basic geometric relationships embedded within the figure.

In propositional design, Stage One features abundant explicit given conditions relative to its limited inferential requirements. The diagram contains clear geometric markers including fixed angles, parallel lines, midpoints, and congruent line segments, and students only need one or two simple reasoning steps to derive a straightforward conclusion. This design lowers the threshold for student participation, establishes standardized geometric language and

fundamental relational premises for later tiers, and lays out the core geometric configuration to be explored in depth. A typical propositional task at this stage involves verifying basic geometric equalities such as triangle congruence, whose proven results become the essential logical premise for Stages Two and Three.

3.2 Stage Two: Relational Application and Deductive Reasoning

Stage Two imposes moderate cognitive demands under both theoretical frameworks. Per Bloom's taxonomy, it primarily targets the "applying" and "analyzing" tiers; per the Van Hiele model, it falls at Level 3 (informal deduction) and occasionally Level 4 (formal axiomatic proof). Its distinctive propositional characteristic lies in introducing incremental local constraints, most commonly movable geometric elements, variable parameters, or additional geometric rules. Students must deduce implicit relational properties unobservable in the static base figure from Stage One.

Instead of examining fixed geometric facts, Stage Two centers on relational reasoning. Learners must integrate conclusions derived in Stage One with newly added constraints to draw non-trivial logical inferences. Common reasoning tasks include angle chasing, similarity ratio derivation, area equation construction, and trigonometric correlation analysis. Functionally, Stage Two acts as the logical core of the three-tier structure: it requires students to deploy Stage One's proven results as foundational premises for new deductions, distinguishing learners who genuinely grasp geometric concepts from those who merely memorize mechanical problem-solving procedures. It therefore serves as an effective screening tool to differentiate conceptual comprehension from superficial procedural fluency.

3.3 Stage Three: Comprehensive Synthesis and Open-Ended Exploration

The third stage is at the highest level of both frameworks: it falls under the 'evaluate and create' categories in Bloom's revised taxonomy and is considered a Level 4 or 5 according to Van Hiele's model. The propositional function is to have students use all the results from the previous steps to construct an argument that can either establish a general relationship, optimize a geometric quantity, construct a new object under certain conditions, or prove a claim that is not obvious from the earlier sub-questions.

Stage Three usually adds a touch of generalisation, optimisation or conditional branching; that is, it determines the range of a geometric quantity with respect to a parameter, identifies all positions of a movable point that meet a given constraint, or proves a result that holds for a more general class of configurations. Therefore, metacognition is required; one should think not only about geometric facts but also about how to organise the arguments for open-ended questions. The discrimination of Stage Three is not due to geometric novelty but rather because the solver needs to construct a whole line of reasoning, not just perform operations.

4. Propositional Logic in Item Construction

4.1 The Logic of Condition Setting

The reason for the three-stage progressive geometry item is not arbitrary; rather, in terms of propositional design, it has been selected reasonably to determine what can and cannot be deduced at each step. Item designers need to distinguish between boundary conditions, which determine the general shape of the whole and are the same in all three stages; and stage-local conditions, which are added one by one as each new sub-question is introduced.

Good boundary conditions should be relatively rich in terms of the three stages of inference but also relatively simple so as to be shown in one easy-to-understand figure. Specify the type of shape, main connections of parts such as parallelism, perpendicularity, or tangency, and fixed numbers to serve as the basis for concrete calculation. Only when necessary will there be parametric variations or additional constraints at the stage level, based on the principle of minimal new information; that is to say, each stage adds only the conditions needed to answer its own question and does not disclose information that would make a later stage obvious. Stage Two is too easy; therefore, it will not serve to differentiate between students' learning abilities effectively.

4.2 The Logic of Scaffolded Questioning

Each subquestion contains a defined propositional task that specifies the target conclusion and implicitly restricts the scope of viable reasoning approaches. Silver [5] noted that the framing of mathematical problems directly shapes students' motivation for mathematical inquiry. In three-stage progressive items, each question articulates a clear immediate inferential objective while retaining flexibility for multiple independent solution paths, without mandating a

single fixed solving method.

A well-designed Stage One question requires only one straightforward deductive path to reach its conclusion. A well-designed Stage Two question centers on relational modeling: students must express geometric quantities as functions of variable parameters or derive fixed relationships amid structural transformations, with multiple partial reasoning approaches available. A well-designed Stage Three question demands open-ended integrated analysis, requiring exhaustive case classification or optimization reasoning to determine geometric value ranges or characterize all valid configurations. This ordered progression of question types forms the item's logical narrative arc, with inferential demands escalating gradually in alignment with the natural trajectory of geometric exploration.

4.3 Dependency Relations and Coherence

The propositional coherence of the three stages of a progressive item is based on their dependency structure. An item has coherence if, at every step, the question is not only more difficult than the one before it but also directly involves the results or methods obtained in that previous step. When this dependency is properly calibrated, the item will form an authentic propositional scaffold; students who can complete Stage One will have obtained the necessary tools for Stage Two, and those who can finish Stage Two will have acquired the tools required for Stage Three.

Polya believed that in solving a mathematical problem, one needs to build a logical bridge connecting the initial state of the problem and the goal state. The propositional dependency structure of the three-stage progressive items is used to achieve this; that is to say, each sub-question is both a goal state that serves as the target of the question and a given state that provides resources for the next one. The fluent switching between the two roles in the three stages is an impressive cognitive achievement and is not merely about one geometric skill.

5. Principles for Effective Item Design

5.1 Cognitive Load Management

The primary challenge of designing three-stage progressive geometry items lies in regulating students' cognitive load across all tiers. Developers must continuously adjust intrinsic load—the inherent difficulty of geometric reasoning—and minimize extraneous load introduced through poor visual presentation. The unified geometric diagram should incorporate all elements required for the three tiers without redundant distracting components. Consistent symbolic labeling must be maintained across all subquestions, allowing students to reuse annotations and geometric relationships identified in Stages One and Two without reconstructing the entire geometric context from scratch for Stage Three.

5.2 Alignment with Curriculum Standards

The three stages of the progressive geometry items should be in line with the cognitive goals and content standards of the official mathematics curriculum. The Mathematics Curriculum Standards for Compulsory Education (2022 Edition) [7] specifically require the cultivation of geometric intuition, logical reasoning, mathematical modelling and creative thinking. The first stage of a good three-part question is to sense one's intuition and basic reasoning; the second stage is to build a mathematical model by altering parameters; and the third stage is open-ended and needs creative synthesis. As shown by the results of PISA 2022 [8], the ability to construct, use, and judge mathematical arguments is now one of the most frequently examined indicators of international mathematical literacy, which is consistent with the three-stage format.

5.3 Differentiation and Assessment Validity

A typical problem in standardised tests is that we need to have many items that can distinguish students with various levels of ability and at the same time be valid indicators of the desired abilities. The three-stage progressive format has increased the gradient of cognitive demand in its construction to address this; Stage 1 is easy for all students, Stage 2 can differentiate students with above-average grades from those with lower grades, and Stage 3 is for excellent students. Crucially, this differentiation is motivated by the nature of the increase in cognitive demand; that is to say, an increase in the difficulty of geometric reasoning requires an actual improvement in its depth and flexibility, not merely an increase in the number of calculations, so it can reliably assess the depth and adaptability of students' geometric reasoning.

6. Conclusion

Systematic deconstruction of the propositional logic in the three-stage progressive geometry comprehensive examination items has been conducted in this paper, and due to its high relevance, it is gradually being applied to Chinese secondary school mathematics assessments. Based on Bloom's revised taxonomy and Van Hiele's model of geometric thinking, according to the analysis, there are three stages in the propositional scaffold, and the cognitive demand is increased by providing a rule for sharing information and inferential dependence.

The Analysis has drawn some particular conclusions. First, the three stages are at different levels of geometric reasoning: Stage One is basic recognition and recall; Stage Two is relational application and formal deduction; and Stage Three is comprehensive synthesis and open-ended exploration. Second, the propositional logic of such items is based on the three overlapping Design logics: setting conditions to determine the speed of geometric information display; scaffolded questioning to organise the level of inferential demands in a graded way from simple to complex; and propositional dependency to ensure that each stage is a necessary logical foundation for the next. Thirdly, good Design should be able to handle cognitive load reasonably well, align with the curriculum, have an attractive motivation factor, create a difficulty gradation, and support both differentiation and evaluation.

These results will be applied by mathematics teachers and in the construction of tests. Item designers should start with a propositional map of the three-stage sequence, clarify the dependency relationships first, and then build the geometric figure; that is to say, establish the logical architecture before detailing the surface geometry. Classroom teachers can use the three-stage structure of student assessment to provide them with instruction and lead students to study independently in the final stage through deduction. In the future, empirical studies will be carried out to investigate how students manage propositional dependency structures at different stages of logical transfer between stages, and how dynamic geometry software environments can improve the transparency of deductive logic for learners and promote the development of reflective and self-aware mathematical reasoning.

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