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Solving Practical Problems Using the Similarity of Triangles as a Motivation for Learning

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Abstract

Since geometry contains basic tools for students' mathematical work, it is of indisputable importance to get students to appropriate the basic concepts of this branch of Mathematics, and to also integrate these concepts into geometry as a whole. Geometry contributes to the development of visualization, critical thinking, problem solving, hypothesis management, logical argumentation, and students' skills in demonstration; it also provides powerful tools to represent and solve problems in all areas of Mathematics, other school disciplines, and real-life problems. The theoretical foundation of this work rests on the use of mathematical language as a means of representing mathematical objects, the transformations of semiotic registers, and the conception of Mathematics as a means and object. This research was carried out through case studies, using action research methods. With the aim of arriving at a set of didactic-methodological guidelines to support teachers in teaching the subject.

Keywords

Geometry; similarity; semiotic representation; mathematical tools

1. Introduction

Since Mathematics is built with mathematical tools, each concept that the student incorporates into their mathematical knowledge is an instrument to study new mathematical objects [1]. Students are more interested in the practical use they can give to what they have learned; it is a situation that should be taken advantage of in the teaching-learning process of Mathematics. However, although what is proposed is a useful resource, it is not enough; the teacher must master, or at least understand, the ontological and epistemological characteristics of Mathematics so that he can develop an efficient direction of said process [2].

Thus, we have that the non-ostensive nature of mathematical objects has implications in their epistemology and consequently in their didactics, since by not having the object itself, it is necessary to resort to its semiotic representations, but each representation only highlights certain characteristics of the object of study, so it is necessary to resort to changes in semiotic representation registers. In the work [3], an analysis is made of the influence of changes in representation registers for the study of the similarity of triangles.

Another important aspect that must be worked on is the use of language. It is important to explore students' communication in mathematics classes, mainly as a contribution to learning. The individual cognitive change that mathematical learning entails is recognized when one learns to do and speak in the ways legitimized at school and in a certain social structure [4]. Promoting communication in the classroom gives students the opportunity to organize, explore, and clarify their thoughts. The level of understanding of a concept or idea is closely related to the efficient communication of that concept or idea, and that communication improves understanding, just as understanding improves communication. For [5], it is important to explore language during classes, as it has both the function of mediating the learning process, through communication between students and between students and the teacher, as well as an "internal mental function" (p. 102), transforming what is communicated into the internal discourse that organizes thought. These aspects are necessary for conceptual appropriation, understanding that a student has appropriated a concept when he has incorporated it into his cognitive structure and can use it as a tool in solving other problems.

2. Development

Geometry is an important branch of mathematics, and its teaching should therefore be given special attention. Geometry contributes to the development of visualization, critical thinking, problem solving, hypothesis management, logical argumentation, and demonstration skills in students; it also provides powerful tools for representing and solving problems in all areas of mathematics, in other school disciplines, and in real-life problems [6].

Triangles are among the basic shapes of plane geometry, and are frequently found in everyday life. The concept of a triangle is basic in the teaching of geometry and is required for the teaching of more complex geometric concepts. By examining triangles, it is possible to gain information about other polygons.

The topic of similarity is one of the concepts that students have difficulty understanding and learning. Equality and similarity of triangles is one of the most important topics in geometry teaching, not only because we often encounter examples of these in daily life, but also because they are involved in the development of many geometric tasks and are among the basic figures of geometry. The minimum conditions required for two triangles to be similar, side angle side, (S.A.L.), side, side, side (l.l.l.) and angle, angle (A.A.) can be considered simple, however, students are not very successful in identifying similar triangles, according to [7, 8], students often have difficulties in questions in which similar triangles are superimposed or mixed with other figures.

3. Theoretical Bases

The work of R. Duval on the changes of semiotic registers, taken up by many specialists, is fundamental in the work on the similarity of triangles. Duval [9] pointed out that Mathematical objects cannot be perceived directly, so access to them is mediated by the use of representations. Hence, the role of semiotic representation systems. beyond a simple means of labelling mathematical objects, but rather they allow a person to work on and with mathematical objects.

Duval does not only refer to isolated representations, but to systems of representations that allow transformations of representations to be made within a representation system without changing the mathematical object being represented. Duval referred to these systems of representations as registers and specified two different types of transformations of semiotic representations that can take place during any mathematical activity, namely, treatments and conversions. Treatments involve transformations from one semiotic representation to another within the same system or register, while conversions involve changing the system being worked on while maintaining reference to the same objects [10].

4. Another Aspect to Take into Account

4.1 The use of mathematical language

Collaborative activities, in which a productive exchange of learning takes place, will have, as a cross-cutting axis, effective communication as a powerful mechanism to negotiate mathematical meanings and develop thinking strategies, fundamental skills to continue learning. Consequently, starting from the fact that the mechanism through which we share ideas, problems, and opinions is language, it should be obvious that mathematical ideas, concepts, and problems are also transmitted through a language that we call technical and whose ignorance is usually the cause of greater difficulties in learning mathematics [4, 11]. The correct command of mathematical language guarantees social interaction, a fundamental aspect for student learning, according to [5].

Furthermore, language is one of the first forms of materialization of mathematical objects, but if the student does not have an adequate command of mathematical language, he will have problems interpreting these materializations, much less being able to carry them out himself, and consequently, he will have difficulties communicating with his peers.

Reasoning at complex cognitive levels through mathematical discourse is not something that many students can easily achieve. This is often due to the interference of everyday language within the mathematical register [12].

4.2 Mathematics as a means and object

The specialized bibliography argues for the need to carry out activities where the learner has to articulate the new with the already known and can reflect on the link between the new and the already known.

However, didactic considerations aside, the foundation of the link between the known and the unknown is due to one of the ontological characteristics of Mathematics, that is, the fact that Mathematics is a means and an object in itself; this means that the resources to learn Mathematics are in Mathematics itself; this is something that students frequently underestimate and do not consider each thing learned as a tool that they will need to learn new things within Mathematics and its applications [1].

Many times, in order to ensure that their student have good teaching results, at least in appearance, they avoid giving their students tasks where they have to use resources learned in previous subjects, which clearly leads students to think that the content they have passed is no longer needed. This clearly affects the development of students as they do not have the mathematical tools to work in this discipline.

4.3 Didactical approach

It is based on placing the student at the center of his or her own learning, the social interaction between them, and the use of previous knowledge to solve the proposed tasks. With the use of mathematical assistants such as GeoGebra, Maple, Derive, etc., at the discretion of the teachers.

5. Methodology

This research was carried out using the case study method, which is one of the qualitative research approaches. It can be said that the purpose of case studies is to evaluate a situation, see and identify the factors that make a situation occur, and develop possible explanations about a situation [13]. Action research was also applied since some of the researchers were involved in the development of the research process.

6. Didactic Proposal

As already stated, language plays a fundamental role, and mathematical language in particular, which is different from everyday language. In the case at hand, this is the case. In colloquial language, considering two things similar may depend on personal criteria, but in Mathematics, the similarity of triangles is subject to specific rules, which students must master.

Therefore, the first thing is for students to learn the criteria of similarity, which must be achieved through direct examples, with the triangles in the same position, to move on to propose cases of similar triangles in different positions, as illustrated below:

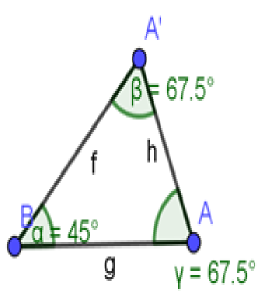


Figure 1. Position 1.

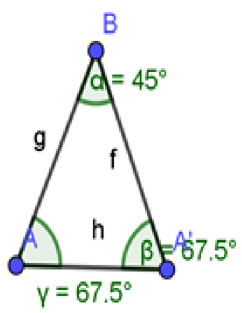


Figure 2. Position 2.

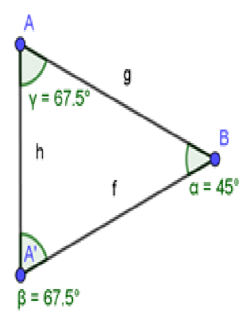


Figure 3. Position 3.

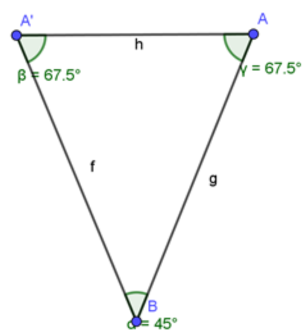


Figure 4. Position 4.

A resource that helps students to separate the similarity from the equality of triangles is the following: a triangle is built in GeoGebra from an angle with a fixed measurement, the triangle can then be moved by the point at the vertex of the angle of fixed measurement and you can see how the position of the triangle changes as well as the lengths of its sides while maintaining the condition of similarity:

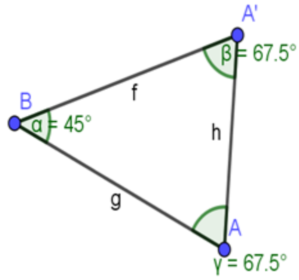
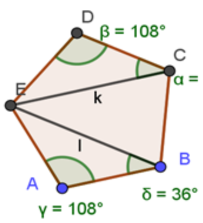
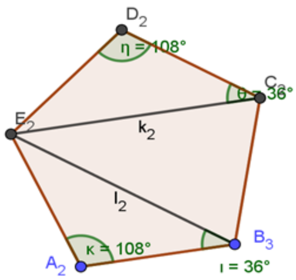


Figure 5. Different side lengths.

This support with dynamic geometry figures contributes significantly to students' internalizing the concept of similarity. With this help, students will not have major difficulties in identifying similar triangles when they are presented in the same position and not mixed with other figures; therefore, it is necessary to train students to identify similarity in composite figures, for example:





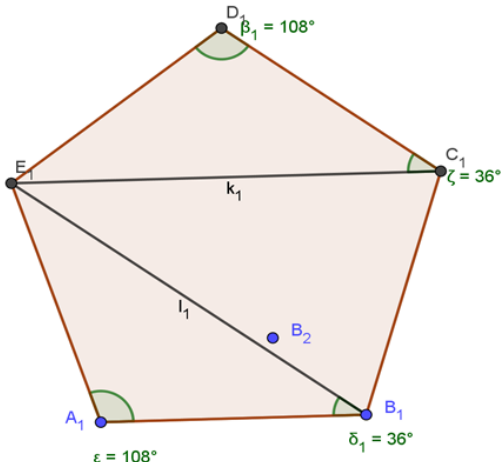


Figure 6. Composite figure 1. Figure 7. Composite figure 2. Figure 8. Composite figure 3.

It is also necessary to set tasks for students where they have to use their geometric knowledge to determine whether or not the characteristics that allow ensuring the similarity of triangles exist, as in the following examples:

If CD is parallel to EF, identify the similar triangles on the circle with center A.

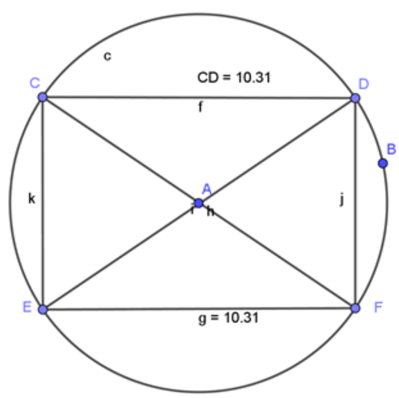


Figure 9. Determine the similarity.

In the following figure, determine whether triangles ABC and EFC are similar, justifying the similarity criterion used:

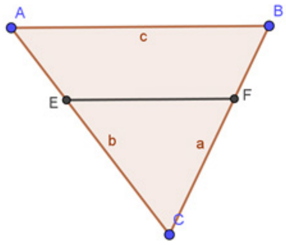
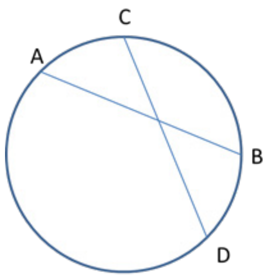


Figure 10. Apply similarity criteria.

There is a circle with center at point O, and a fixed point P inside said circle, AC and BD are any two chords of the same circle that contain said point P. In which cases does AP/BP=DP/CP?



Hacer las construcciones auxiliares
AD y CB

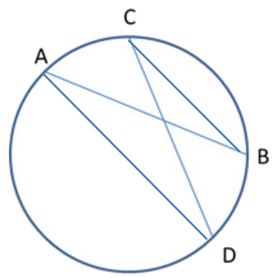


Figure 11. Application of similarity. **Figure 12. Application of similarity.**

Triangles APD and CPB are similar, the angles at P opposite the vertex, angles ADC and ABC, subtend the same arc, and the same with angles DAB and BCD. The proportionality of homologous sides of similar triangles gives the requested result. Of course, in this type of task, it is possible that the students' problem is not in their knowledge of similarity, but in their knowledge of the necessary geometric relations; however, this type of task is necessary to emphasize Mathematics as a means and an object. In addition, exercises with different degrees of complexity can be found to adjust the task to the students' level.

As is the case in the following example, determining the required similarity is more immediate.

We have the similar triangles ABC and A´B´C´ so the ratios of their homologous sides are equal: AB / A'B' = BC / B'C' = CA / C'A' = k. Find the ratio BH / B'H' between the lengths of the heights of the triangles.

It is important for students to be able to transfer the statement in the literal register to its representation in the geometric register, since it is necessary to train students in changing representation registers.

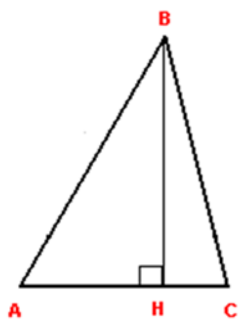


Figure 13. Change of registration. Figure14. Change of registration.

Angles A and A´ are equal because they are homologous angles in similar triangles, and angles H and H´ are equal because they are determined by the height, so triangles ABH and A´B´H´ are similar and therefore: AB / A'B' = BH / B'H' = k

The similarity of triangles is generally used to determine the ratio between their homologous sides, but to consolidate students' understanding of the topic, tasks are also necessary where they have to determine whether the triangles are similar based on the ratio between their sides, as in the following cases:

a) 40, 30, 50 & 120, 90, 150 b) 10, 5, 15 & 6, 3, 9 c) 40, 60, 70 & 6, 9, 10

d) 3, 9, 3 & 20, 60, 20.

Examples such as the one in Section d should not be missing, where, although the three pairs have the same ratio, they do not form triangles because the sum of two is less than the third.

Although to learn Mathematics nothing else is needed apart from Mathematics, as a motivation for students and also to consolidate their knowledge, practical application tasks such as those shown are necessary:

You can start with simple examples and within geometry itself, such as the following:

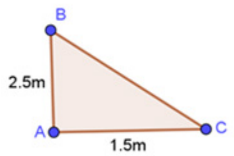
The legs of a right triangle measure 24 m and 10 m. How long will the legs of a triangle similar to the first one be, whose hypotenuse measures 52 m?

As you can see, the statement itself indicates the application of the similarity of triangles.

Examples of practical applications, such as the following, should also be proposed:

A 2.5-meter vertical pole casts a 1.5-meter shadow. Determine the height of a vertical tree, which at the same time casts a 6-meter shadow.

In this case, since the problem is given in a literal representation, it is appropriate to make a change of register and use a figure semiotics as shown:



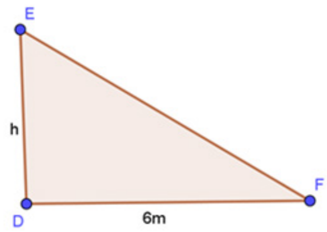


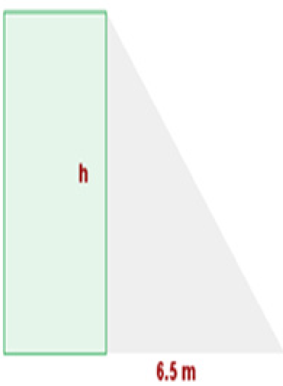
Figure 15. Practical application. Figure 16. Practical application.

Note that BC and EF are parallel because they are projections of the same light at the same time, and AC and DF are also on a straight line, considering that the pose and the tree have their base at the same level; therefore, triangles ABC and DEF are similar.

Therefore: $\frac{2.5}{h} = \frac{1.5}{6}$ then $h = \frac{2.5\times 6}{1.5} = 10$ and the height of the tree is 10 m.

Calculate the height of a building that casts a shadow of 6.5 m at the same time that a 4.5 m high pole casts a shadow of 0.90 m.

As in the previous case, a representation in the Figure record illustrates the steps to follow. Students should be encouraged to change the representation themselves.



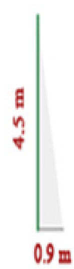


Figure 17. Practical application. Figure18. Practical application.

Since the shadows are projected at the same time, we can assume the similarity in order to provide a solution. Thus, given the similarity, we obtain the following equality:

$\frac{4.5}{h} = \frac{0.9}{6.5}$, despejando h, se obtiene $h = \frac{(6.5)(4.5)}{0.9} = 32.5$

7. Didactic and Methodological Guidelines

In teaching the topic of triangle similarities, in addition to the content itself, it is necessary to emphasize the need for students to use mathematical language appropriately, using the word "similarity" to illustrate the need for mathematical language, taking into account that it is not only a means of communication, but also an internal mental function [15].

Another important aspect to take into account is the non-ostensive nature of mathematical objects, which requires the need for changes in semiotic representation.

It is essential to insist to students that each rule or property they learn is a new work tool that they must have ready to use to solve new problems or tasks.

The introduction to the topic should begin by differentiating between similarity in colloquial language and mathematical language, highlighting that in mathematical language, one cannot work with implicit understandings, and introducing the requirements that two triangles must meet to be similar. This introduction should be accompanied by representations of similar and dissimilar triangles using mathematical assistants such as GeoGebra, Maple, Derive, or others, continuing with composite figures where similar triangles appear.

Continue with tasks where the similarity criteria are not immediate, but are derived from the statements, continue with tasks where it is necessary to use previous knowledge to determine the conditions of similarity, with the objective of emphasizing Mathematics as a means and object.

Finally, propose tasks of practical problems that are solved through the similarity of triangles and other geometric knowledge. These tasks, which should play a motivating role for the students, which in general should be presented literally, so that the students practice the changes of representation from literal records to geometric records. These tasks close the cycle of appropriation of the concepts in Mathematics, of actions on the objects, which are developed in processes that identify the objects as concepts, which should be incorporated into the cognitive structure of the subject, according to the APOS theory (action, process, object, scheme) of Ed Dubinsky [14].

The central idea of these guidelines is to place the student at the center of his or her own learning, promoting social interaction between students, with tasks that require the use of prior knowledge to solve said tasks. Supported by the use of mathematical assistants such as GeoGebra, Maple, Derive, etc., at the discretion of the teachers.

8. Conclusions

As explained, the topic is conducive to showing students fundamental differences between everyday language and mathematical language, given that similarity in everyday language may be criterion-based, but in mathematical language, it is subject to specific norms; that is, in mathematical language, there are no implicit understandings, and each concept obeys fixed and well-defined norms.

It also demonstrates the need for changes in representational registers, particularly from literal to figurative or analytical registers, which once again demonstrates the importance of proper use of mathematical language, since only with a good understanding of the latter can one move from literal language to another.

Another particularly important aspect addressed in this work is the approach to mathematics as both a medium and an object, so that students internalize that each mathematical concept they learn is a tool they will need to learn new concepts. This requires developing a teaching-learning process with students responsible for their own learning.

It is also emphasized that every opportunity should be taken to guide students so they can practice making scientific generalizations, that is, based on the essential features of objects and phenomena. All of this allowed us to arrive at the didactic-methodological guidelines for teaching the topic of triangle similarity.

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