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Analysis and Control of the Pressurized Water Nuclear Reactor Model

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Abstract

The nuclear materials produced by pressurized water nuclear reactors are vital for advanced cancer treatments, particularly through the creation of radioisotopes. The dynamics of the PWR are very complex and nonlinear, and it is important to understand the nonlinearity and develop effective control mechanisms. In this work, bifurcation analysis and multiobjective nonlinear model predictive control are performed on the pressurized water nuclear reactor model. Bifurcation analysis is a powerful mathematical tool used to deal with the nonlinear dynamics of any process. Several factors must be considered, and multiple objectives must be met simultaneously. The MATLAB program MATCONT was used to perform the bifurcation analysis. The MNLMPC calculations were performed using the optimization language PYOMO in conjunction with the state-of-the-art global optimization solvers IPOPT and BARON. The bifurcation analysis revealed the existence of Hopf bifurcation points, limit points, and branch points. The MNLMC converged on the Utopian solution. The Hopf bifurcation point, which causes an unwanted limit cycle, is eliminated using an activation factor involving the tanh function. The limit and branch points (which cause multiple steady-state solutions from a singular point) are very beneficial because they enable the Multiobjective nonlinear model predictive control calculations to converge to the Utopia point (the best possible solution) in the model.

Keywords

Cancer; Tumor; Bifurcation; Optimization; Control; Pressurized water nuclear reactor

1. Background

The pressurized water nuclear reactor is one of the most widely used nuclear reactor designs in the world and represents a major milestone in the development of nuclear energy technology. Its design, operation, and widespread application have been driven by the demand for reliable, efficient, and safe production of electricity, as well as by the desire for stable power sources to complement fossil fuels and renewable energy. At its core, the pressurized water reactor functions on the principle of using pressurized light water both as a coolant and as a neutron moderator, which allows the reactor to sustain a controlled nuclear fission chain reaction while efficiently transferring the heat produced to generate electricity.

The basic structure of a pressurized water reactor involves a reactor vessel that houses the nuclear fuel, control rods, and coolant. The fuel typically consists of uranium dioxide pellets that are encased in zirconium alloy cladding to form fuel rods. These rods are bundled into fuel assemblies that are arranged in a precise geometry inside the

reactor core. When the reactor operates, the uranium-235 or other fissile material undergoes fission when struck by a thermal neutron, splitting into smaller nuclei and releasing additional neutrons and a large amount of heat. The neutrons produced from fission are moderated, or slowed down, by the light water in the reactor core, which makes it more likely for them to cause further fission events. This moderation is essential to maintain a self-sustaining chain reaction at a controlled rate.

The control rods, made of neutron-absorbing materials such as boron, silver, indium, or cadmium, are inserted or withdrawn from the reactor core to regulate the neutron flux and therefore control the power output. By absorbing excess neutrons, they provide a mechanism to slow down or shut off the chain reaction if necessary. This feature is fundamental to reactor safety, as it allows operators to adjust reactivity levels and respond to changes in demand or abnormal operating conditions.

A distinctive feature of the pressurized water reactor, compared to other reactor types, is that the primary coolant loop is kept under very high pressure, typically around 150 atmospheres. This high pressure prevents the water from boiling even at temperatures of around 320 degrees Celsius. Because the coolant remains in the liquid state, it can effectively transfer heat from the reactor core without undergoing a phase change. The heated primary coolant circulates through closed loops and transfers its thermal energy to a secondary loop via steam generators. In the steam generator, the heat from the primary coolant is used to boil water in the secondary circuit, producing steam that drives turbines connected to electrical generators. After driving the turbines, the steam is condensed back into water in a condenser, and the cycle repeats.

This arrangement of primary and secondary coolant loops provides an additional layer of safety because the radioactive materials in the primary coolant are kept separate from the secondary loop and the turbine system. Only the primary loop is in direct contact with the reactor core, so the secondary loop remains largely free from radioactive contamination, making the turbine and associated systems easier to maintain and safer to operate.

The design of the pressurized water reactor emphasizes both efficiency and safety. One of its inherent safety features is the negative temperature coefficient of reactivity. This means that as the temperature of the reactor core increases, the probability of neutron absorption by the coolant also increases, and the overall reactivity of the reactor decreases. In other words, the reactor naturally tends to stabilize itself as the temperature rises, reducing the risk of runaway reactions. This characteristic has been crucial in the widespread adoption of PWR technology.

In addition to the physical and operational characteristics, the development of pressurized water reactors has been shaped by historical, political, and economic factors. The technology was first developed in the United States in the 1950s, initially for use in naval propulsion systems for submarines. The success of this design in military applications demonstrated its reliability and safety, which later facilitated its adaptation for commercial power generation. By the 1960s and 1970s, pressurized water reactors had become the dominant reactor design for civilian nuclear power plants in many countries. Their widespread deployment was supported by standardization, strong safety records, and the growing need for reliable electricity production during periods of industrial expansion.

Today, pressurized water reactors account for the majority of operational nuclear power plants around the globe. Countries such as the United States, France, China, Russia, South Korea, and Japan have large fleets of PWRs providing a significant share of their electricity. France, in particular, relies heavily on nuclear power, with most of its reactors based on the pressurized water design. The reliability of these reactors has been instrumental in making France one of the countries with the lowest carbon dioxide emissions per capita in the developed world, demonstrating the role of PWRs in reducing greenhouse gas emissions.

Safety has always been a central concern in nuclear power, and the pressurized water reactor has been designed with multiple redundant safety systems. Emergency core cooling systems are available to supply coolant in case of a loss-of-coolant accident. Containment structures, consisting of thick reinforced concrete and steel liners, are built around the reactor vessel to prevent the release of radioactive materials into the environment in case of accidents. Passive safety systems have also been integrated into more modern PWR designs, using natural circulation and gravity-fed cooling rather than active pumps and external power sources.

Despite these safety features, the operation of PWRs is not without risks or challenges. Accidents such as the Three Mile Island incident in 1979 highlighted the importance of human factors, operator training, and robust safety culture. Although the accident resulted in only minor releases of radioactivity and no direct casualties, it had significant impacts on public perception of nuclear power, regulatory frameworks, and reactor design. Lessons from Three Mile Island, as well as from later accidents such as Chernobyl and Fukushima, have led to continuous improvements

in safety systems, emergency preparedness, and reactor technology.

Another challenge associated with pressurized water reactors is the management of spent nuclear fuel. After several years of operation, fuel assemblies become less efficient and are replaced with fresh fuel. The spent fuel remains highly radioactive and generates heat, requiring careful handling, cooling, and long-term storage. Interim storage is typically carried out in spent fuel pools, while some countries have developed dry cask storage systems for longer-term management. The issue of permanent disposal of nuclear waste remains unresolved in many countries, though geological repositories are being planned or developed in some regions.

Economically, pressurized water reactors have both advantages and drawbacks. On one hand, they provide large-scale, continuous, and low-carbon electricity, which contributes to energy security and helps mitigate climate change. On the other hand, the construction of nuclear power plants requires significant capital investment, long lead times, and regulatory approval processes that can be costly and time-consuming. Maintenance, decommissioning, and waste management add further costs over the lifetime of a plant. These economic challenges, along with public concerns about nuclear safety, have influenced the pace of new reactor construction in recent decades.

Nonetheless, innovation in PWR technology continues. Advances such as small modular reactors (SMRs) based on PWR designs are being developed to address concerns about cost, scalability, and flexibility. These smaller reactors are designed for easier construction, enhanced safety, and the potential for deployment in remote areas or smaller grids. Digital control systems, improved materials for fuel cladding, and more efficient steam generators are also being incorporated into modern PWR designs. International collaboration has facilitated the exchange of knowledge and best practices, ensuring that pressurized water reactors remain a cornerstone of nuclear energy for decades to come.

The environmental impact of PWRs is generally favorable when compared to fossil fuel power plants. They produce virtually no greenhouse gas emissions during operation and have relatively small land footprints compared to renewable sources like solar and wind. However, the thermal pollution from cooling water discharges and the issue of radioactive waste management remain environmental concerns. In many cases, nuclear plants have implemented cooling towers and closed-cycle cooling systems to minimize impacts on local ecosystems.

Looking to the future, pressurized water reactors will continue to play a significant role in global energy strategies. As the world seeks to balance energy demand, environmental sustainability, and energy security, nuclear power offers a reliable low-carbon option. PWRs, with their proven track record and adaptability, are likely to remain a dominant design, though they may be complemented by advanced reactors and a growing share of renewable energy sources. Public acceptance and political support will be critical in determining the extent to which nuclear power, and PWRs in particular, can contribute to addressing the dual challenges of climate change and energy security.

The pressurized water reactor represents a highly developed, safe, and efficient technology that has profoundly influenced the landscape of nuclear energy. From its origins in naval propulsion to its dominance in commercial power generation, the PWR has demonstrated reliability and adaptability. While challenges related to safety, waste management, and economics remain, continuous innovation and lessons learned from past experiences have strengthened its role as a key energy source. As societies grapple with the urgent need for low-carbon, large-scale, and stable power supplies, the pressurized water nuclear reactor stands as both a technological achievement and a continuing subject of research, development, and debate in the pursuit of sustainable energy futures.

Van Bragt studied the effect of void distribution parameter and axial power profile on BWR bifurcation characteristics [1]. Dokhane et al. performed bifurcation analysis of density wave oscillations using a drift flux model [2]. Zhou et al. conducted bifurcation analysis of nuclear coupled dynamics of BWRs using BIFDD [3]. Dokhane et al. developed a nuclear-coupled thermal-hydraulic nonlinear stability analysis using a novel BWR reduced-order model [4, 5]. Zhou et al. investigated the dynamics of a reduced-order natural circulation BWR model [6].

Zhou et al. studied the in-phase and out-of-phase oscillations in BWRs, investigating the impact of Azimuthal asymmetry [7]. Merk et al. developed multiple timescale expansions for neutron kinetics [8, 9]. Rizwan-uddin demonstrated the existence of turning points and sub- and supercritical bifurcations in a simple BWR model [10]. Gabor et al. modelled a nuclear reactor with Temperature Effects and Xenon Poisoning [11]. Wahi et al. conducted a nonlinear stability analysis of a reduced-order model of nuclear reactors, performing a parametric study relevant to the advanced heavy water reactor [12]. Pirayesh et al. studied local bifurcations in nuclear reactor dynamics by Sotomayor's Theorem [13].

In this work, bifurcation analysis and multiobjective nonlinear model predictive control are performed on a

pressurized water nuclear reactor model [13]. The paper is organized as follows. First, the model equations are presented, followed by a discussion of the numerical techniques involving bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC). The results and discussion are then presented, followed by the conclusions

2. Model Equations

The model equations are

$$\begin{aligned}\frac{d(pv)}{dt} &= \frac{pv}{\lambda c}(\alpha f(uv) + \alpha c(wv) + c0 - \beta) + \frac{\beta cv}{\lambda c}\\ \frac{d(cv)}{dt} &= \lambda s(pv - cv)\\ \frac{d(uv)}{dt} &= a1(pv) - a2(uv - wv + \phi 1)\\ \frac{d(wv)}{dt} &= a3(uv - wv + \phi 1) - a4(wv + \phi 2)\end{aligned} \tag{1}$$

The base parameter values are

a1=95.057; a2=0.251; a3=0.092; a4=2.749; $\phi 1$ 1=395; λc =0.00002;β =0.006019;
λs =0.15; αf =-0.0000324;αc =-0.000213;$\phi 2$ =15; c0=0.004.

pv, cv, uv, and wv represent the relative power of a reactor, the relative concentration of delayed neutron precursors, the fuel temperature, and the coolant temperature. λc is the effective generation time of neutrons, c0 is the reactivity inserted into the reactor core, β is the delayed neutrons fraction, λs is the effective precursor radioactive decay constant, αf is the reactivity coefficient of fuel temperature, αc is the reactivity coefficient of coolant temperature. a1, a2, a3, a4, are the thermo- hydraulics coefficients.

3. Bifurcation Analysis

Bifurcation analysis deals with multiple steady-states (caused by branch and limit points) and limit cycles, which are caused by Hopf bifurcation points. The MATLAB program MATCONT [14, 15] is used to locate limit points, branch points, and Hopf bifurcation points. In the ODE system

$$\frac{dx}{dt} = f(x, \alpha) \tag{2}$$

$x \in R^n$ Let the bifurcation parameter be α. Since the gradient is orthogonal to the tangent vector,
The tangent plane is the n+1-dimensional vector w that satisfies

$$Aw = 0 \tag{3}$$

Where A is

$$A = [\partial f/\partial x \quad |\partial f/\partial \alpha] \tag{4}$$

And $\partial f/\partial x$ is the Jacobian matrix. For both limit and branch points, the Jacobian matrix $J = [\partial f/\partial x]$ must be singular.

For a limit point, there is only one tangent at the point of singularity. At this singular point, there is a single non-zero vector, y, where Jy=0. This vector is of dimension n. Since there is only one tangent the vector
$y = (y_1, y_2, y_3, y_4, \ldots y_n)$ must align with $\widehat{w} = (w_1, w_2, w_3, w_4, \ldots w_n)$. Since

$$J\widehat{w} = Aw = 0 \tag{5}$$

the n+1th component of the tangent vector w_{n+1} = 0 at a limit point (LP).

For a branch point, there must exist two tangents at the singularity. Let the two tangents be z and w. This implies that

$$\begin{aligned}Az &= 0\\ Aw &= 0\end{aligned} \tag{6}$$

Consider a vector v that is orthogonal to one of the tangents (say w). v can be expressed as a linear combination of z and w ($v = \alpha z + \beta w$). Since $Az = Aw = 0$; $Av = 0$ and since w and v are orthogonal,

$w^T v = 0$. Hence $Bv = \begin{bmatrix} A \\ w^T \end{bmatrix} v = 0$ which implies that B is singular.

Hence, the matrix $B = \begin{bmatrix} A \\ w^T \end{bmatrix}$ is singular at a branch point.

When there is a Hopf bifurcation point the bialternate product,

$$det(\, 2f_x(x, \alpha) @ Jn) = 0 \tag{7}$$

where Jn is the n-square identity matrix. Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. More details can be found in Kuznetsov [16, 17] and Govaerts [18].

Hopf bifurcations cause limit cycles. Limit cycles cause equipment damage and make control tasks difficult. Additionally, they result in less beneficial products. The tanh activation function (where a control value v is modified to $(v \tanh v / \varepsilon)$ is used to eliminate spikes in profiles [19-22]. Sridhar [23] demonstrates with several examples how the activation factor involving the tanh function successfully eliminates the limit cycle causing Hopf bifurcation points by increasing the oscillation time period in the limit cycle.

4. Multiobjective Nonlinear Model Predictive Control (MNLMPC)

The rigorous multiobjective nonlinear model predictive control (MNLMPC) method developed by Flores Tlacuahuaz et al. was used [24]. Consider a problem where the variables $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ have to be optimized together for a dynamic problem

$$\frac{dx}{dt} = F(x, u) \tag{8}$$

t_f being the final time value and u the control parameter. The individual objective optimal control problem is solved by optimizing each of the variables $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ The optimization of $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ will lead to the values q_j^*. Then the multiobjective optimal control (MOOC)problem $min(\sum_{j=1}^{n} \left(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^*\right))^2$

$$subject\ to\ \ \frac{dx}{dt} = F(x, u); \tag{9}$$

is solved. This will provide the value of u at each time step. The first obtained control value of u is implemented and this procedure is repeated until the implemented and the first obtained control values are the same or where $(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^* = 0$ for all j. Utopia point) is obtained.

The optimization program PYOMO is used [25]. Here, the differential equations are converted to a Nonlinear Program (NLP) using the orthogonal collocation method. The NLP is solved using IPOPT [26] and confirmed as a global solution with BARON [27].

The steps of the algorithm are as follows

1. Optimize $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ and obtain q_j^*.

2. Minimize $(\sum_{j=1}^{n}(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^*))^2$and get the control values at various times.

3. Implement the first obtained control values

4. Repeat steps 1 to 3 until there is an insignificant difference between the implemented and the first obtained value of the control variables or if the Utopia point is achieved. The Utopia point is when $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^* = 0$ for all j.

Sridhar demonstrated that when the bifurcation analysis revealed the presence of limit and branch points the MNLMPC calculations to converge to the Utopia solution [28]. For this, the singularity condition, caused by the presence of the limit or branch points was imposed on the co-state equation [29]. If the minimization of q_1 lead to the value q_1^* and the minimization of q_2 lead to the value q_2^* The MNLPMC calculations will minimize the function $(q_1 - q_1^*)^2 + (q_2 - q_2^*)^2$. The multiobjective optimal control problem is

$$min \quad (q_1 - q_1^*)^2 + (q_2 - q_2^*)^2 \quad subject\ to \quad \frac{dx}{dt} = F(x, u) \tag{10}$$

Differentiating the objective function results in

$$\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) = 2(q_1 - q_1^*)\frac{d}{dx_i}(q_1 - q_1^*) + 2(q_2 - q_2^*)\frac{d}{dx_i}(q_2 - q_2^*) \quad (11)$$

The Utopia point requires that both $(q_1 - q_1^*)$ and $(q_2 - q_2^*)$ are zero. Hence

$$\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) = 0 \quad (12)$$

The optimal control co-state equation [33] is

$$\frac{d}{dt}(\lambda_i) = -\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) - f_x\lambda_i; \quad \lambda_i(t_f) = 0 \quad (13)$$

λ_i is the Lagrangian multiplier. t_f is the final time. The first term in this equation is 0 and hence

$$\frac{d}{dt}(\lambda_i) = -f_x\lambda_i; \lambda_i(t_f) = 0 \quad (14)$$

At a limit or a branch point, for the set of ODE $\frac{dx}{dt} = f(x,u)$ f_x is singular. Hence there are two different vectors-values for $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) > 0$ and $\frac{d}{dt}(\lambda_i) < 0$. In between there is a vector $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) = 0$. This coupled with the boundary condition $\lambda_i(t_f) = 0$ will lead to $[\lambda_i] = 0$ This makes the problem an unconstrained optimization problem, and the optimal solution is the Utopia solution.

5. Results and Discussion

The bifurcation analysis was performed with λs as bifurcation parameter, reveals a branch point, a Hopf bifurcation point, and two limit points at (1.331485, 1.331485, 111.126353, 1.875616, 0); (1.331485, 1.331485, 111.126353, 1.875616, 169.591848); (1.331446, 1.331317, 111.111412, 1.875132, 0); and (1.331741, 1.332614, 111.226724, 1.878866, 0). Figure 1 shows the bifurcation diagram. Figure 2 shows the limit cycle that occurs because of the Hopf bifurcation point. When λs is changed to $\lambda s\, tanh(\lambda s)/10$ the Hopf bifurcation disappears (see Figure 3).

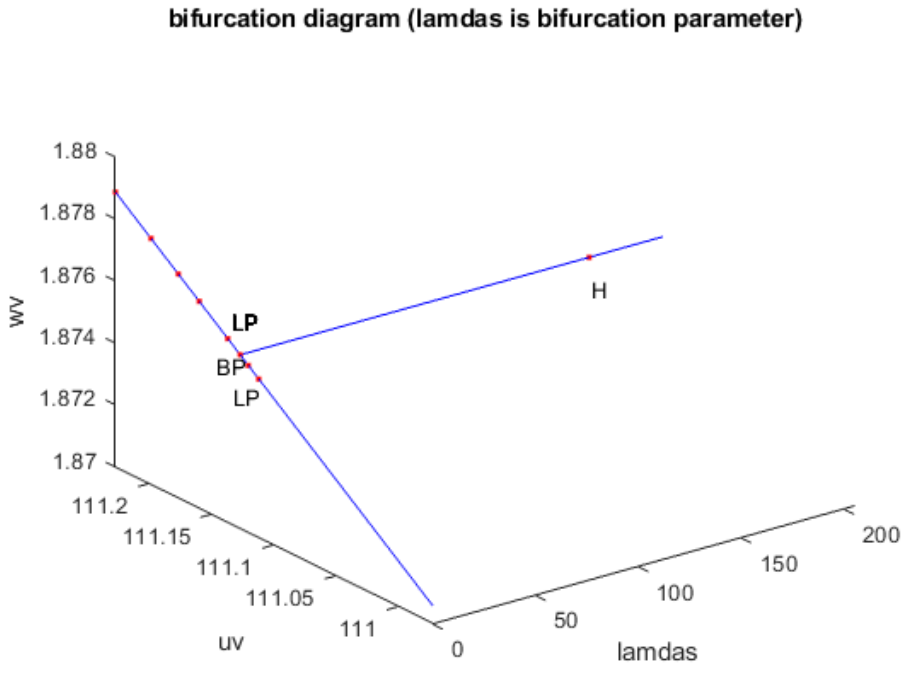

Figure 1. Bifurcation diagram (λs is the bifurcation parameter).

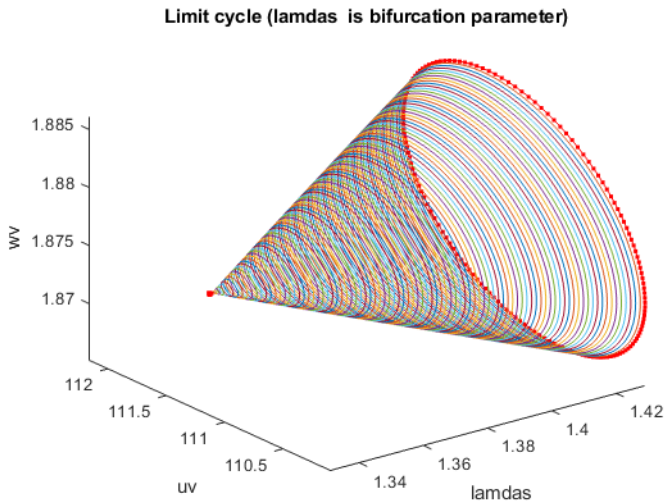

Figure 2. Limit cycle (λs is the bifurcation parameter).

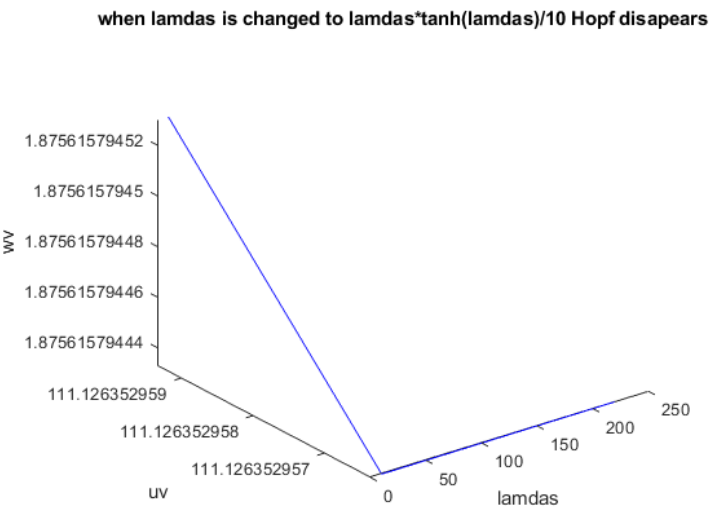

Figure 3. When λs is changed to $\lambda s\, tanh(\lambda s)/10$ Hopf bifurcation disappears.

In the study of nuclear reactor dynamics, the stability of the system is of paramount importance because even small instabilities can lead to potentially dangerous operating conditions or costly shutdowns. A key mathematical concept that often arises in nonlinear dynamical systems is the Hopf bifurcation, which occurs when a stable equilibrium loses stability as system parameters change and a stable or unstable limit cycle emerges. In the context of pressurized water nuclear reactors, Hopf bifurcations can represent the onset of self-sustained oscillations in the neutron population, temperature fields, or coolant flow variables. These oscillations are problematic because they can reduce reactor safety margins, complicate control strategies, and increase the risk of mechanical or thermal fatigue within the system. Understanding why Hopf bifurcations are dangerous in pressurized water reactors requires examining the interplay between neutron kinetics, thermal-hydraulic feedback mechanisms, and the operational environment of the reactor.

Pressurized water reactors operate under carefully balanced conditions where nuclear fission generates heat that is transferred to coolant water, which remains under high pressure to prevent boiling in the core. The power level of the reactor is regulated through a combination of control rods and soluble boron concentrations, while the removal of heat is governed by pumps and heat exchangers. This system is inherently nonlinear due to the feedback between neutron flux, fuel temperature, coolant density, and reactivity. In steady-state operation, the reactor is designed to

remain near an equilibrium point where power generation and heat removal balance each other. However, changes in parameters such as coolant flow rate, delayed neutron fractions, reactivity feedback coefficients, or control system gains can alter stability. If these changes push the reactor dynamics past a critical threshold, a Hopf bifurcation can occur, resulting in self-sustained oscillations instead of stable equilibrium behavior.

Such oscillations are problematic for several reasons. First, they can manifest as power oscillations in the neutron flux. Unlike random fluctuations, which are usually small and quickly damped, limit cycles associated with Hopf bifurcations persist indefinitely and maintain finite amplitude. This means that the reactor could experience sustained oscillations in core power, leading to cyclic heating and cooling of fuel rods. Thermal cycling places stress on reactor materials and increases the risk of fatigue or even failure over long-term operation. Pressurized water reactors are designed with strict thermal limits, and persistent oscillations could cause local "hot spots" where temperatures exceed safe thresholds, damaging cladding or creating the conditions for fuel failure.

Second, Hopf bifurcations in pressurized water reactors can couple with thermal-hydraulic instabilities. In PWRs, the moderator and coolant are the same fluid, and its density strongly influences reactivity through the moderator temperature coefficient. If oscillations occur in coolant temperature or density, they feed back into reactivity, reinforcing the oscillatory behavior. This feedback loop can be particularly dangerous because it may amplify otherwise small oscillations into large, self-sustaining ones. For example, a small oscillation in coolant temperature might change neutron moderation efficiency, which alters reactivity, changes power, increases or decreases fuel temperature, and feeds back into coolant conditions. This closed loop is exactly the type of nonlinear dynamic structure where Hopf bifurcations are possible.

Third, control systems designed for PWRs assume a reasonably stable equilibrium operating point. Automatic power control systems, feedwater regulation, and pressurizer pressure controls all rely on damping out small deviations from setpoints. If the system undergoes a Hopf bifurcation, the control system is no longer operating near equilibrium but is instead responding to sustained oscillations. In such a case, conventional proportional-integral-derivative (PID) controllers may be ineffective, or worse, may exacerbate the oscillations if improperly tuned. This means that a Hopf bifurcation undermines the fundamental assumption on which reactor control is based. Without careful nonlinear control design, the reactor may not be able to maintain stable operation, forcing a shutdown to preserve safety.

The occurrence of Hopf bifurcations also complicates reactor analysis and safety assessments. Linear stability analysis, which is widely used in reactor safety studies, focuses on whether eigenvalues of the linearized system lie in the left half of the complex plane. However, Hopf bifurcations involve eigenvalues crossing the imaginary axis, which indicates the onset of oscillations but does not by itself reveal whether the resulting oscillations are stable limit cycles or unstable ones. If the bifurcation is supercritical, the reactor may experience stable, bounded oscillations. While still problematic, these may not be immediately catastrophic, but can degrade safety margins and reliability. If the bifurcation is subcritical, however, unstable limit cycles can emerge, meaning that even small perturbations may drive the reactor into large-amplitude oscillations, potentially leading to unsafe conditions very rapidly. This distinction is subtle but important in understanding why Hopf bifurcations represent a serious concern in nuclear engineering.

Moreover, Hopf bifurcations highlight the sensitivity of PWRs to parameter variations. In practice, parameters such as coolant flow, delayed neutron fractions, or fuel temperature coefficients are not constant but vary with burnup, temperature, and operating conditions. As the reactor ages, material properties change, and coolant chemistry evolves. These parameter changes can inadvertently shift the reactor closer to the bifurcation threshold. This means that a reactor deemed stable under design conditions may become susceptible to oscillations later in its life cycle unless operators and designers account for such possibilities. Long-term operation without acknowledging the risk of Hopf bifurcations could therefore compromise reactor safety margins.

Another problem associated with Hopf bifurcations in pressurized water reactors is the difficulty of detection and diagnosis. Because oscillations associated with limit cycles can be relatively small at first, they may resemble normal fluctuations in power or coolant flow. By the time oscillations grow large enough to clearly indicate a bifurcation has occurred, significant stress may already have been imposed on reactor structures. Furthermore, standard monitoring systems may not be tuned to detect oscillatory instabilities, especially if they fall outside the frequency range of expected noise. This creates a situation where Hopf bifurcations can occur insidiously, slowly degrading system stability until a clear safety concern arises.

From an operational standpoint, Hopf bifurcations are also problematic because they limit flexibility. Nuclear reactors are often required to adjust power levels in response to grid demand. If the reactor is near a Hopf bifurcation point, rapid power maneuvers or changes in coolant flow could inadvertently trigger oscillations. This would reduce the ability of operators to respond flexibly to external requirements and might necessitate overly conservative operation, reducing the efficiency and economic competitiveness of nuclear power generation.

Hopf bifurcations also underscore the challenges of coupling between reactor physics and thermal-hydraulics. In PWRs, these two aspects are inseparable. Neutron dynamics affect heat generation, which affects coolant conditions, which affect reactivity. A Hopf bifurcation in this coupled system cannot be analyzed from a purely neutronic or purely thermal-hydraulic perspective. Instead, it emerges from their nonlinear interplay. This makes predicting and preventing Hopf bifurcations computationally challenging, requiring sophisticated nonlinear dynamic models, bifurcation analysis tools, and simulations that capture the full feedback structure of the reactor.

Finally, Hopf bifurcations matter in the broader context of nuclear safety philosophy. The principle of defense-in-depth requires multiple layers of safety to prevent accidents, mitigate consequences, and protect the public. However, Hopf bifurcations represent an internal, systemic instability rather than an external initiating event. This means they challenge safety at a fundamental level, making it harder to defend against them using traditional layered barriers. If oscillations become large, they may stress components in ways not accounted for in design-basis accidents, potentially invalidating assumptions in safety analysis reports.

Hopf bifurcations are problematic in pressurized water nuclear reactors because they represent a transition from stable equilibrium to self-sustained oscillations, undermining reactor stability, safety, and controllability. They lead to persistent power oscillations, amplify thermal-hydraulic instabilities, compromise control systems, and challenge conventional safety analysis approaches. They are difficult to detect, sensitive to parameter changes, and dangerous if left unaddressed. Most importantly, they emerge from the inherent nonlinear coupling between neutronics and thermal-hydraulics, which lies at the heart of reactor operation. For all these reasons, avoiding Hopf bifurcations and ensuring sufficient margin from oscillatory instabilities remain critical priorities in the design, operation, and analysis of pressurized water reactors.

The use of the tanh activation factor eliminates the Hopf bifurcation point, validating the hypothesis in Sridhar [23].

For the MNLMPC c0 is the control parameter, and $\sum_{t_i=0}^{t_i=t_f} pv(t_i)$, $\sum_{t_i=0}^{t_i=t_f} cv(t_i)$ were maximized individually, and each of them led to values of 10 and 9.88007.

The overall optimal control problem will involve the minimization of $(\sum_{t_i=0}^{t_i=t_f} pv(t_i) - 10)^2 + (\sum_{t_i=0}^{t_i=t_f} cv(t_i) - 9.0887)^2$ was minimized subject to the equations governing the model. This led to a value of zero (the Utopia point). The MNLMPC value of the control variable, c0 was 0.02688. The branch and limit points cause the MNLMPC calculations to attain the Utopia solution, validating the analysis of Sridhar [28]. The MNLMPC profiles are shown in Figs 4-7.

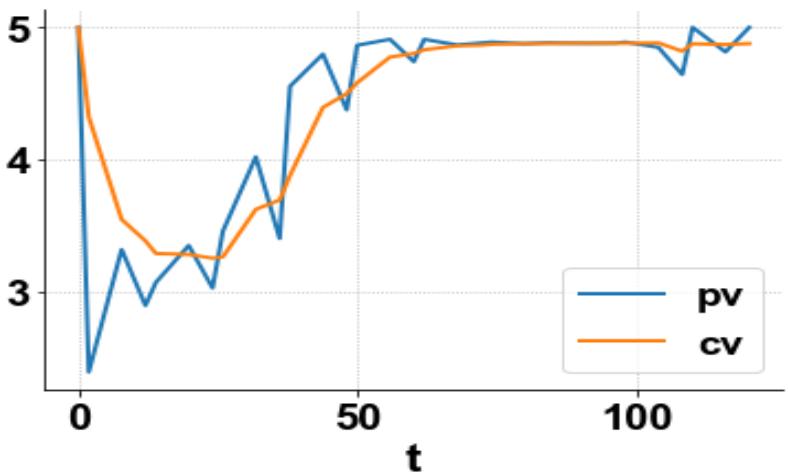

Figure 4. MNLMPC (pv, cv profiles).

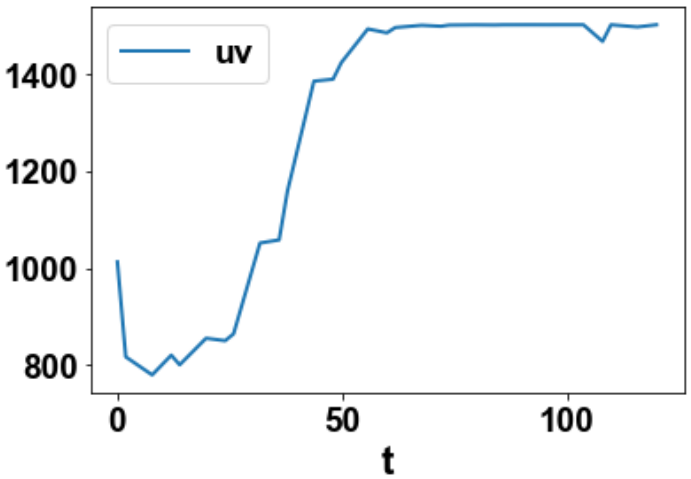

Figure 5. MNLMPC (uv profile).

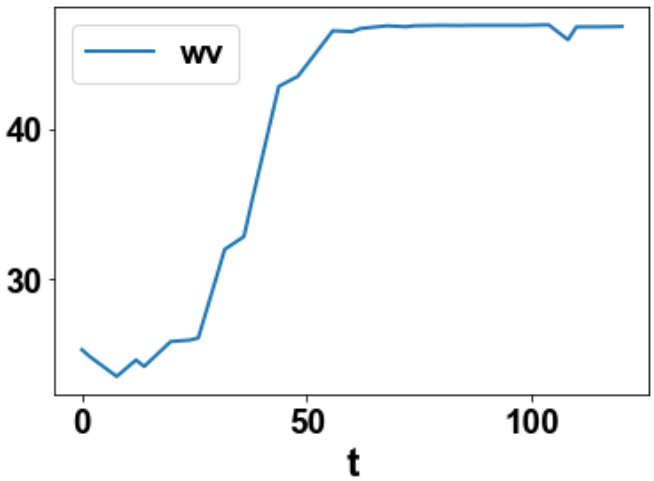

Figure 6. MNLMPC (wv profile).

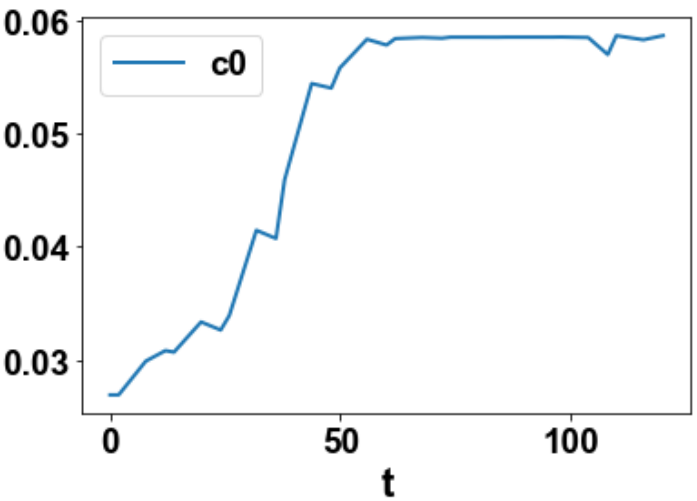

Figure 7. MNLMPC (c0 profile).

6. Conclusions

Bifurcation analysis and multiobjective nonlinear control (MNLMPC) studies on the pressurized water nuclear reactor model. The bifurcation analysis revealed the existence of Hopf bifurcation points, limit points, and branch points. The Hopf bifurcation point, which causes an unwanted limit cycle, is eliminated using an activation factor involving the tanh function. The limit and branch points (which cause multiple steady-state solutions from a singular point) are very beneficial because they enable the Multiobjective nonlinear model predictive control calculations to converge to the Utopia point (the best possible solution) in the models. A combination of bifurcation analysis and Multiobjective Nonlinear Model Predictive Control (MNLMPC) for the pressurized water nuclear reactor model is the main contribution of this paper.

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Data Availability Statement

All data used is presented in the paper.

Conflict of Interest

The author has no conflict of interest to declare.

References

[1] Van Bragt DDB, Rizwan-uddin, van Der Hagen THJJ. Effect of void distribution parameter and axial power profile on BWR bifurcation characteristics. Nucl Sci Eng. 2000;134:227-35.

[2] Dokhane A, Hennig D, Rizwan-uddin, Chawla R. Bifurcation analysis of density wave oscillations using a drift flux model. In: Proceedings of the PHYSOR-2002 International Conference on the New Frontiers of Nuclear Technology: Reactor Physics, Safety and High-performance Computing; 2002 Oct 7-10; Seoul, Korea. LaGrange Park, IL: American Nuclear Society; 2002.

[3] Zhou Q, Rizwan-uddin. Bifurcation analysis of nuclear coupled dynamics of BWRs using BIFDD. In: Proceedings of the PHYSOR-2002 International Conference on the New Frontiers of Nuclear Technology: Reactor Physics, Safety and High Performance Computing; 2002 Oct 7-10; Seoul, Korea. LaGrange Park, IL: American Nuclear Society; 2002.

[4] Dokhane A, Hennig D, Chawla R, Rizwan-uddin. Nuclear-coupled thermal-hydraulic nonlinear stability analysis using a novel BWR reduced order model. Part 1. The effects of using drift flux versus homogeneous equilibrium models. In: Proceedings of the International Conference on Nuclear Engineering (ICONE-11); 2003 Apr 20-23; Tokyo, Japan.

[5] Dokhane A, Hennig D, Chawla R, Rizwan-uddin. Nuclear-coupled thermal-hydraulic nonlinear stability analysis using a novel BWR reduced order model. Part 2. Stability limits of in-phase and out-of-phase modes. In: Proceedings of the International Conference on Nuclear Engineering (ICONE-11); 2003 Apr 20-23; Tokyo, Japan.

[6] Zhou Q, Rizwan-uddin. Dynamics of a reduced order natural circulation BWR model. In: Proceedings of the PHYSOR 2004 - The Physics of Fuel Cycles and Advanced Nuclear Systems: Global Developments; 2004 Apr 25-29; Chicago, IL. LaGrange Park, IL: American Nuclear Society; 2004.

[7] Zhou Q, Rizwan-uddin. In-phase and out-of-phase oscillations in BWRs: impact of azimuthal asymmetry and second pair of eigenvalues. Nucl Sci Eng. 2005;151:95-113.

[8] Merk BR, Cacuci DG. Multiple timescale expansions for neutron kinetics-II: illustrative application to P1 and P3 equations. Nucl Sci Eng. 2005;151:184-93.

[9] Merk BR, Cacuci DG. Multiple timescale expansions for neutron kinetics-II: illustrative application to P1 and P3 equations. Nucl Sci Eng. 2005;151:194-211.

[10] Rizwan-uddin. Turning points and sub- and supercritical bifurcations in a simple BWR model. Nucl Eng Des. 2006;236(3):267-83.

[11] Gabor A, Fazekas C, Szederkényi G, Hangos K. Modeling and identification of a nuclear reactor with temperature effects and xenon poisoning. In: 2009 35th Annual Conference of IEEE Industrial Electronics (IECON); 2009 Nov 3-5; Porto, Portugal. IEEE; 2009.

[12] Wahi P, Kumawat V. Nonlinear stability analysis of a reduced order model of nuclear reactors: a parametric study relevant to the advanced heavy water reactor. Nucl Eng Des. 2011;241(1):134-43.

[13] Pirayesh B, Pazirandeh A, Akbari M. Local bifurcation analysis in nuclear reactor dynamics by Sotomayor's theorem. Ann Nucl

Energy. 2016;94:716-31.

[14] Dhooge A, Govaerts W, Kuznetsov YA. MATCONT: a MATLAB package for numerical bifurcation analysis of ODEs. ACM Trans Math Softw. 2003;29(2):141-64.

[15] Dhooge A, Govaerts W, Kuznetsov YA, Mestrom W, Riet AM. CL_MATCONT: a continuation toolbox in MATLAB. In: Proceedings of the 2004 ACM Symposium on Applied Computing; 2004 Mar 14-17; Nicosia, Cyprus. New York: ACM; 2004. p. 1614-8.

[16] Kuznetsov YA. Elements of applied bifurcation theory. 2nd ed. New York: Springer-Verlag; 1998.

[17] Kuznetsov YA. Five lectures on numerical bifurcation analysis. Utrecht, Netherlands: Utrecht University; 2009.

[18] Govaerts WJF. Numerical methods for bifurcations of dynamical equilibria. Philadelphia: Society for Industrial and Applied Mathematics; 2000.

[19] Dubey SR, Singh SK, Chaudhuri BB. Activation functions in deep learning: a comprehensive survey and benchmark. Neurocomputing. 2022;503:92-108.

[20] Kamalov AF, Nazir M, Safaraliev A, Cherukuri R, Zgheib R. Comparative analysis of activation functions in neural networks. In: 2021 28th IEEE International Conference on Electronics, Circuits, and Systems (ICECS); 2021 Nov 28-Dec 1; Dubai, United Arab Emirates. IEEE; 2021. p. 1-6.

[21] Szandała T. Review and comparison of commonly used activation functions for deep neural networks. In: Biologically inspired cognitive architectures 2019. Singapore: Springer; 2020. p. 203-24.

[22] Sridhar LN. Bifurcation analysis and optimal control of the tumor macrophage interactions. Biomed J Sci Tech Res. 2023;53(5):45123-8.

[23] Sridhar LN. Elimination of oscillation causing Hopf bifurcations in engineering problems. J Appl Math. 2024;2(4):1826.

[24] Flores-Tlacuahuac A, Morales P, Rivera-Toledo M. Multiobjective nonlinear model predictive control of a class of chemical reactors. Ind Eng Chem Res. 2012;51(17):5891-9.

[25] Hart WE, Laird CD, Watson JP, Woodruff DL, Hackebeil GA, Nicholson BL, et al. Pyomo - optimization modeling in Python. 2nd ed. New York: Springer; 2017.

[26] Wächter A, Biegler LT. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. Math Program. 2006;106(1):25-57.

[27] Tawarmalani M, Sahinidis NV. A polyhedral branch-and-cut approach to global optimization. Math Program. 2005;103(2):225-49.

[28] Sridhar LN. Coupling bifurcation analysis and multiobjective nonlinear model predictive control. Austin Chem Eng. 2024;10(3):1107.

[29] Upreti SR. Optimal control for chemical engineers. Boca Raton: CRC Press; 2013.