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Discrete States in Atoms as a Consequence of Maxwell's Equations and the Theory of Relativity

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Abstract

Six years before Schrödinger presented his equation (1925), on October 28, 1919, Max Planck, posed the first and most difficult dilemma facing the whole of quantum theory whether light rays are themselves quantized or whether quantum effects occur only in matter. In this article of mine, it is assumed that quantum actions occur in matter, that is, that quantum actions do not exist by themselves, but that quantum effects occur within matter. My starting point for the assumption that quantum effects occur in matter is an examination of Planck's dilemma mentioned above, with a focus on matter, and relying on the clear equations of Maxwell and the Special theory of relativity. The results are understandable and can be understood better than those starting from the Schrödinger equation. Although the path presented in this article is more difficult, it is more exact and can be proven, unlike the path taken by Schrödinger, when he wrote his equation without a clear starting point. The aim of this study is to define the structural constant of all atoms s_0 and the action of *LC* oscillator *A*, as a new concept. The methods of theoretical research are used, and it's checking is based on previously measured data. Electromagnetic radiation, which we observe in an area outside of the atom, has its source in the atom. As a model of this source an *LC* oscillator was investigated within the atom. It is determined that the energy of that *LC* oscillator is proportional to its natural frequency. However, the proportionality factor A, which is analogous to Planck's *h*, is not constant, but changes with the change of its natural frequency. Periodic coincidence of two independent phenomena within an atom is condition of the stability of an atom. These two phenomena are, first, circulating the electron around the nucleus, and second, oscillating the eletromagnetic energy in the atom. At the integer frequency ratio of these two phenomena, discretization of the atoms state occurs. The structural constant s_0 and its unified value 8.278691910 is defined. All NIST data, from Hydrogen, $_1$H, to Darmstadtium, $_{110}$Ds, at least 110 different measurements, confirmed this value. This approach, besides the atomic shell, includes its nucleus. It is shown that with help of structural constant s_0, as well with help of the other five known constants (c, μ_0, e, m, m_p), nine existing constants become interchangeable; i.e., fine structure constant, von Klitzing constant R_K, Planck's h, ratio e/h, Josephson constant K_J, Rydberg constant R_∞, Bohr radius a_0, Bohr magneton μ_B, and nuclear magneton μ_N. All relevant physical quantities are also given in a form suitable for use in Discrete Physics. All relations in Discrete Physics are as clear as in Classical Physics. Planck's $h = A$, defined as the ratio of the energy of a photon to its frequency, is not a constant. The structural constant of the atom s_0 is unchanging. Also constant is $h_0 = A_0$, but defined as $h_0 = A_0 = \mu_0 c\, e^2 s_0^2$ = 6.627 882 313 × 10-34 J· Hz^{-1}, and this should be adopted as the true value of Planck's constant, which is 0.0273% greater than its NIST-recognized value (6.626 070 15×10-34 J·Hz^{-1}).

Keywords

Maxwell equations; Theory of relativity; Structural constant; Discrete physics

1. Introduction

At the very beginning it should be noted that the theory presented here is based on Maxwell's electromagnetism and on the Special theory of relativity. Maxwell's equations for free space in their differential form are [2] (p.742):

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}, \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{3}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \tag{4}$$

In the previous equations, E and B are the electric and magnetic field strengths, respectively, while ρ and $\mathbf{J}$ are the charge densities and electric current densities, respectively. The operator ∇ is the del operator, [2] (p. 740), defined in Cartesian coordinates as

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}. \tag{5}$$

The quantity is called the divergence of E,

$$\nabla \cdot \mathbf{E} = \frac{\partial Ex}{\partial x} + \frac{\partial Ey}{\partial y} + \frac{\partial Ez}{\partial z}. \tag{6}$$

According to Maxwell's equations, an electric charge emits electromagnetic energy whenever it accelerates. Thus, an electron orbiting the nucleus of an atom, with constant centripetal acceleration, constantly emits electromagnetic energy. Therefore, its energy should gradually decrease. This would eventually lead to a gradual decrease in the dimensions of its orbit, and the electron would finally fall into the nucleus. However, at the same time, there is a process in the atom that occurs in the opposite direction. According to these same laws, an electron in an atom also absorbs electromagnetic radiation. Namely, it is the radiation reaction force that, by emitting electromagnetic radiation in a stationary state, contributes to the absorption of electromagnetic radiation in the atom [3]. This means that in the stationary state of the atom, this absorption is equal to the emission of electromagnetic radiation, so the atom remains stable.

2. Atom as a Transmission Line and LC Oscillator

We will consider that the source of the electromagnetic wave is a structure of the substance that it consists of central body with a charge *Q*, while around this body at a distance *r* (orbit radius), with uniform velocity **v** in circular motion revolving the body with mass *m* and with the charge *q*, that's the Bohr model [2] (p. 880-85). Unlike Nils Bohr, here we consider a model in which the source of electromagnetic radiation is an *LC* oscillator, which is obtained from the transmission line (*Lecher line*, Figure 1), consisting of pair of ideal conductive nonmagnetic wires [4] (59, 359-60) located in a sphere of radius *r* placed inside an insulated sphere of radius *r*, its vacuum permittivity is ε_0=8.854 187 8188(14) × 10^{-12} $\mathrm{F \cdot m^{-1}}$ (farad per meter) and its magnetic permeability is $\mu_0 = 4 \cdot \pi \cdot 10^{-7}$ $\mathrm{H \cdot m^{-1}}$ (henry per meter). Considering the properties of such a transmission line (ideal conductive nonmagnetic wires), such a line can be mathematically processed independently of its possible physical implementation. This form of transmission line and oscillator can be mathematically imagined and described independently of their physical realization. Therefore, the transmission line does not physically exist within the atom, but its mathematical model is used. just as mathematical models in space exploration work well without celestial bodies involved.)

The *angular frequency* ω is $2\pi f$, so the *natural frequency* f of this *LC* oscillator is [2] (p. 700):

$$f = 1/(2\pi\sqrt{LC}). \tag{7}$$

So, as mentioned, the source of the electromagnetic wave is a structure of the substance that it consists of a central body with a charge *Q*, while around this central body in a steady state at the distance *r*, with uniform speed *v* in circular motion revolving the body with mass *m* and with charge *q*. This moving body has an acceleration directed radially toward the center the circle, *i.e.*, at right angle to the vector of velocity **v** of magnitude v^2/r [2] (p. 56).

Relativistic momentum **p** of a moving particle is [2] (p. 859):

$$\mathbf{p} = \frac{m\,\mathbf{v}}{\sqrt{1-\beta^2}}, \qquad \boldsymbol{\beta} = \mathbf{v}/c, \quad \mathbf{p} = \frac{m\boldsymbol{\beta}c}{\sqrt{1-\beta^2}}, \tag{8}$$

where **v** is the velocity vector of the motion body, c is the speed of light and m is rest mass of body. The law of conservation of momentum is valid even in the relativistic realm [2] (p. 859). Newton's second law, stated in its most general form

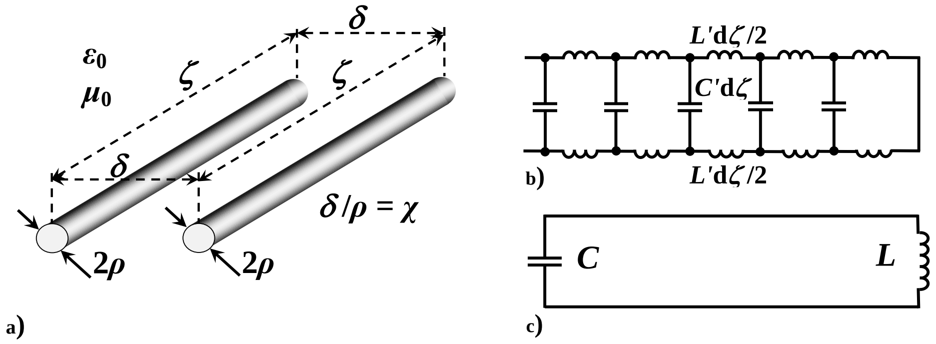

Figure 1. Model.

a) A section of Lecher's line that is long ζ; it is a twin-lead transmission line consisting of pair of ideal conductive nonmagnetic wires of diameter 2ρ, separated by δ, situated in space with permittivity ε_0 and permeability μ_0.

b) Lecher's line presented by an infinite number of extremely small uniformly distributed capacitors, with capacitance $C'\mathrm{d}\zeta$, and with equals such inductors, with inductance $L'\mathrm{d}\zeta$, [4] (p. 59, 359-60).

c) All mentioned capacitors are collected at the open end of the line, denoted by C, and all mentioned inductors are collected on its short-circuited end, and denoted by L, resulting in an LC circuit, placed inside an insulated sphere of radius r, the capacity of that sphere is $C = 4\pi\varepsilon_0 r$, [2] (p. 565-8). Considering the properties of such a transmission line (ideal conductive nonmagnetic wires), I would like to point out that such a line can be mathematically processed independently of its possible physical implementation, which means that the transit line in the atom does not need to be realized but can be quite well described mathematically. The transmission line does not physically exist within the atom, but its mathematical model is used, just as mathematical models in space exploration work well without celestial bodies involved.

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = d\left(\frac{m\,\mathbf{v}}{\sqrt{1-\beta^2}}\right)/dt. \tag{9}$$

also remains valid in relativistic theory. But since the mass of particle cannot be considered constant in relativistic theory Newton second law written as **F**=m**a** is not valid, rather, the variability of mass must be taken into account.

In the stationary state, in circular motion of particle q around particle Q, when variables are at their fixed amounts, Coulomb's attractive force $\frac{qQ}{4\pi\varepsilon_0 r^2}$ [2] (p. 509) and centrifugal force are equalized; we take that q and Q have **opposite** sign and q is **negative**. This means that it is valid:

$$\frac{mv^2}{r\sqrt{1-\beta^2}} = -\frac{qQ}{4\pi\varepsilon_0 r^2}. \tag{10}$$

From Eq. (10) it follows:

$$r = -\frac{qQ}{4\pi\varepsilon_0 mc^2}\frac{\sqrt{1-\beta^2}}{\beta^2}. \tag{11}$$

The kinetic energy of the moving body (particle) is [2] (p. 859-62)

$$K = \frac{mc^2}{\sqrt{1-\beta^2}} - mc^2. \tag{12}$$

To consider the presence of potential energy U in the relativistic case, let's look at the following:

3. Relativistic Charged Particle in an Electromagnetic Field

The relativistic Lagrangian for a particle (rest mass m and charge q) is given by [5], according to the definitions in the electromagnetic field (derivatives of individual quantities are marked with a dot above the respective quantity):

$$\mathcal{L}(\mathrm{t}) = -mc^2\sqrt{1-\frac{\dot{\mathbf{x}}(t)^2}{c^2}} + q\dot{\mathbf{x}}(t)\cdot\mathbf{A}(\mathbf{x}(t),t) - q\varphi(\mathbf{x}(t),t). \tag{13}$$

The particle's canonical momentum is

$$\mathbf{p}(\mathrm{t}) = \frac{\partial \mathcal{L}}{\partial \dot{\mathbf{x}}} = \frac{\dot{\mathbf{x}}(t)}{\sqrt{1-\frac{\dot{\mathbf{x}}(t)^2}{c^2}}} + q\mathbf{A}, \tag{14}$$

that is, the sum of the kinetic momentum and the potential momentum.

Solving for the velocity, we get

$$\dot{\mathbf{x}}(t) = \frac{\mathbf{p} - q\mathbf{A}}{\sqrt{m^2 + \frac{(\mathbf{p}- q\mathbf{A})^2}{c^2}}}. \tag{15}$$

The Hamiltonian is

$$\mathcal{H}(\mathrm{t}) = \dot{\mathbf{x}} \cdot \mathbf{p} - \mathcal{L} = c\sqrt{m^2c^2 + (\mathbf{p} - q\mathbf{A})^2} + q\varphi. \tag{16}$$

This results in the force equation (equivalent to the Euler–Lagrange equation)

$$\dot{\mathbf{p}} = -\frac{\partial \mathcal{H}}{\partial \mathbf{x}} = q\dot{\mathbf{x}} \cdot (\nabla\, \mathbf{A}) - q \nabla \varphi = q \nabla (\dot{\mathbf{x}} \cdot \mathbf{A}) - q \nabla \varphi, \tag{17}$$

from which one can derive

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\dot{\mathbf{x}}}{\sqrt{1-\frac{\dot{\mathbf{x}}}{c^2}}}\right) = \frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{p} - q\mathbf{A}) = \dot{\mathbf{p}} - q\frac{\partial \mathbf{A}}{\partial t} - q(\dot{\mathbf{x}} \cdot \nabla)\mathbf{A}, \tag{18}$$

$$= q \nabla (\dot{\mathbf{x}} \cdot \mathbf{A}) - q \nabla \varphi - \frac{\partial \mathbf{A}}{\partial t} \dot{\mathbf{x}} \cdot \nabla\, \mathbf{A}, \tag{19}$$

$$= q\mathbf{E} + q\dot{\mathbf{x}} \times \mathbf{B}. \tag{20}$$

The above derivation makes use of the vector calculus identity:

$$\frac{1}{2} \nabla (\mathbf{A} \cdot \mathbf{A}) = \mathbf{A} \cdot \mathbf{J_A} = \mathbf{A} \cdot (\nabla\, \mathbf{A}) = (\mathbf{A} \cdot \nabla)\mathbf{A} + \mathbf{A} \times (\nabla\, \mathbf{A}). \tag{21}$$

An equivalent expression for the Hamiltonian as function of the relativistic (kinetic) momentum, $\mathbf{P} = m\dot{\mathbf{x}}(t) = \mathbf{p} - q\mathbf{A}$ is

$$\mathcal{H}(t) = \dot{x} \cdot \mathbf{P}(\mathrm{t}) + \frac{mc^2}{\gamma} + q\varphi(\mathbf{x}(t), t) = \gamma mc^2 + q\varphi(\mathbf{x}(t), t) = E + U, \tag{22}$$

here

$$\gamma = \frac{1}{\sqrt{1-\frac{\dot{\mathbf{x}}^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}}. \tag{23}$$

This has the advantage that kinetic momentum **P** can be measured experimentally whereas canonical momentum **p** cannot. Notice that the Hamiltonian (total energy) can be viewed as the sum of the relativistic energy (kinetic+rest), Eq. (12), plus the potential energy, potential energy U:

$$E = K + mc^2 + U = \frac{mc^2}{\sqrt{1-\beta^2}} + U. \tag{24}$$

Taking into account that the potential φ determined from the Coulomb force $\frac{qQ}{4\pi\varepsilon_0 r^2}$ is equal to $\frac{Q}{4\pi\varepsilon_0 r}$, and taking into account equation (11), the potential energy U is:

$$U = q\varphi = -\frac{mc^2\beta^2}{\sqrt{1-\beta^2}}. \tag{25}$$

Let us now determine the potential φ from $\frac{Q}{4\pi\varepsilon_0 r}$ and from Eq. (11):

$$\varphi = \frac{Q}{4\pi\varepsilon_0 r} = -\frac{mc^2\beta^2}{q\sqrt{1-\beta^2}}. \tag{26}$$

Since the electric potential is defined as the potential energy per unit charge, then the charge in potential energy of a charge q when moved between two points a and b is

$$\Delta U = qV_\mathrm{b} - qV_\mathrm{a} = q\varphi_\mathrm{b} - q\varphi_\mathrm{a} = U_\mathrm{b} - U_\mathrm{a} = qV_\mathrm{ba}. \tag{27}$$

In order for the system to remain stationary (*meaning it no longer emits or absorbs energy*) it has by law of conservation of energy emitted exactly such a large amount of energy $\Delta W = W - W_0 = E_\mathrm{em} = qV_\mathrm{em}$, with the opposite sign between ΔW and E_em; namely, the energy lost by the atom ΔW gets to the emitted energy E_em of the photon. For simplicity, we will take that initial velocity β_0 is always zero, so from the previous expression it follows:

$$E_\mathrm{em} = -W = -qV_\mathrm{em}, \tag{28}$$

which can be related to equations (12) and (25):

$$E_{\rm em} = K + U = \left(\frac{mc^2}{\sqrt{1-\beta^2}} - mc^2\right) + \left(-\frac{mc^2\beta^2}{\sqrt{1-\beta^2}}\right) = -mc^2\frac{-1+\beta^2+\sqrt{1-\beta^2}}{\sqrt{1-\beta^2}} = -mc^2\left(1-\sqrt{1-\beta^2}\right) = -qV_{\rm em}, \quad (29)$$

where $V_{\rm em} = V_{\rm U}$ is the potential difference (voltage) through which passes the body charged with charge q to get the same energy as the electromagnetic energy $E_{\rm em}$ emitted (as we have said, the charge q is negative; $q = -ze$, $z = 1, 2, 3, \dots$, e is elementary charge, while $Q = +Ze$, $Z = 1, 2, 3, \dots$ is positive).

From Eq. (29) we can express $\sqrt{1-\beta^2} = \left(1 - \frac{E_{\rm em}}{mc^2}\right)$ and $\beta^2 = 2E_{\rm em}\frac{\left(1-\frac{E_{\rm em}}{2mc^2}\right)}{mc^2}$, and we include these two in Eq. (11), we get:

$$r = -\frac{qQ}{4\pi\varepsilon_0 mc^2}\frac{\sqrt{1-\beta^2}}{\beta^2} = -\frac{qQ}{8\pi\varepsilon_0 E_{\rm em}}\frac{\left(1-\frac{E_{\rm em}}{mc^2}\right)}{\left(1-\frac{E_{\rm em}}{2mc^2}\right)}, \quad (30)$$

and also:

$$U = q\frac{Q}{4\pi\varepsilon_0 r} = qV_{\rm U} = -\frac{mc^2}{\sqrt{1-\beta^2}}\beta^2 = -\frac{2E_{\rm em}\left(1-\frac{E_{\rm em}}{2mc^2}\right)}{\left(1-\frac{E_{\rm em}}{mc^2}\right)}. \quad (31)$$

$$E_{\rm em} = -\frac{qQ}{8\pi\varepsilon_0 r}\frac{1-\frac{E_{\rm em}}{mc^2}}{1-\frac{E_{\rm em}}{2mc^2}}. \quad (32)$$

Furthermore, with Eq. (29), ($E_{\rm em} = -qV_{\rm em}$, and $\varepsilon_0 = \frac{1}{\mu_0 c^2}$) from Eq. (32) we get:

$$r = \left|\frac{\mu_0 c^2 Q}{8\pi V_{\rm em}}\frac{1-\frac{qV_{\rm em}}{mc^2}}{1-\frac{qV_{\rm em}}{2mc^2}}\right|. \quad (33)$$

So, for example, for the simplest case, for the first orbit, with measured [6] $V_{\rm em}$ = 13.59843449 V, ($q = -ze = -e; Q = Ze = e$), from Eq. (33) we calculate the radius of the first Hydrogen orbit, $r_{\rm H}$ = 5.294526279 × 10^{-11} m, and with $V_{\rm em}$ = 54.417765 V, ($q = -ze = -e;\ Q = Ze = 2e$), we get the radius of the first orbit of Helium, $r_{\rm He}$= 2.645988600 × 10^{-11} m, and with $V_{\rm em}$ = 1362.19915 V, ($q = -ze = -e;\ Q = Ze = 10e$), we get the first orbit of Neon, $r_{\rm Ne}$= 5.278386334 × 10^{-12} m, while for Darmstadtium, $V_{\rm em}$ = 204 400 V, ($q = -ze = -e;\ Q = Ze = 110e$), the first orbit is $r_{\rm Ds}$= 2.905992468 × 10^{-13} m. For multi-electron atoms, it would be necessary to apply Hamilton's equations to point masses systems, which we will not go into here.

The electromagnetic energy $E_{\rm em}$ in the observed structure, which can be an atom too, is the energy of LC oscillator [2] (p. 572, 696-701):

$$E_{\rm em} = \frac{1}{2}\frac{\Theta^2}{C} = \frac{1}{2}LI^2 = \frac{1}{2}L\omega^2\Theta^2. \quad (34)$$

The charge Θ in Eq. (34) is the maximum charge on condenser whose capacitance is $4\pi\varepsilon_0 r$ [2] (p. 565-68), The correlation between the frequency f and the angular frequency ω is Eq. (7), $\omega = 2\pi f = 1/(\sqrt{LC})$

$$C = 4\pi\varepsilon_0 r, \quad (35)$$

and the inductance L, from Eq. (34), and with $\omega = 2\pi f$, is

$$L = \frac{1}{\omega^2 C} = \frac{1}{4\pi^2 f^2 C}. \quad (36)$$

When we equate Eq. (32) and Eq. (34), $\frac{1}{2}\frac{\Theta^2}{C}$, we get:

$$-\frac{1}{2}\frac{qQ}{4\pi\varepsilon_0 r}\frac{(1-\frac{E_{\rm em}}{mc^2})}{(1-\frac{E_{\rm em}}{2mc^2})} = \frac{1}{2}\frac{\Theta^2}{C}. \quad (37)$$

Taking into account equation (34) from Eq. (37) we get:

$$\Theta^2 = -qQ\frac{1-\frac{E_{\rm em}}{mc^2}}{1-\frac{E_{\rm em}}{2mc^2}}. \quad (38)$$

From expressions (35) and (36) we introduce the *characteristic impedance* of an LC circuit as [7], (R = 0 Ω, G = 0 S):

$$Z_{LC} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}} = \sqrt{\frac{\frac{1}{(4\pi^2 f^2 C)}}{C}} = \sqrt{\frac{1}{4\pi^2 f^2 C^2}} = \frac{1}{2\,\pi\, f\, C}. \quad (39)$$

Now we will write Eq. (34) in a different way, by using Eq. (7), Eq. (38) and Eq. (39) to show that the energy of the electromagnetic oscillator $E_{\rm em}$ is proportional to its *natural* (resonant) *frequency* $f = 1/(2\pi\sqrt{LC})$:

$$E_{\rm em} = \frac{1}{2}\frac{\Theta^2}{C} = \frac{1}{2}\frac{\pi}{\pi}\frac{\Theta^2}{\sqrt{C}\sqrt{C}}\frac{\sqrt{L}}{\sqrt{L}} = \pi\sqrt{\frac{L}{C}}\Theta^2\frac{1}{2\pi\sqrt{LC}} = \pi\, Z_{LC}\Theta^2 f = \pi\sqrt{\frac{L}{C}}\left(-qQ\frac{1-\frac{E_{\rm em}}{mc^2}}{1-\frac{E_{\rm em}}{2mc^2}}\right)f = A\, f. \quad (40)$$

It is important to note that although the charge Θ^2, from Eq. (38), participates in Eq. (40) for the energy $E_{\rm em}$, that charge does not participate in the expression for the frequency $f = 1/(2\pi\sqrt{LC})$. Therefore, the electromagnetic energy $E_{\rm em}$, according to Eq. (40), consists of two separate components, one component is dependent on the variables Θ^2, in accordance with equation (38), and the other component is dependent on the fixed parameters of the oscillatory LC circuit, $f = 1/(2\pi\sqrt{LC})$. This will later play an important role in determining the structure constant s_0.

In Eq. (40) A is the *action of the electromagnetic LC oscillator*; it is quotient of electromagnetic energy E_{em} to the natural frequency f of LC oscillator, it is by definition Planck's h:

$$A = \frac{E_{\rm em}}{f} = -\pi\sqrt{\frac{L}{C}}qQ\frac{1-\frac{E_{\rm em}}{mc^2}}{1-\frac{E_{\rm em}}{2mc^2}}\ . \quad (41)$$

The action of the electromagnetic oscillator A is the *proportionality factor* of the electromagnetic energy $E_{\rm em}$ and the natural frequency f of LC circuit. This proportionality factor may or may not necessarily be a constant. All of this depends on the relationships between the electromagnetic energy $E_{\rm em}$ and the natural frequency f, which we will explore below.

Capacitance per unit length C' of Lecher line [4] (p. 61), is:

$$C' = \frac{\pi\,\varepsilon_0}{ln\left(\frac{\chi}{2+\sqrt{\left(\frac{\chi}{2}\right)^2-1}}\right)}, \quad (42)$$

and inductance per unit length L' of Lecher line is [4] (p. 360), is:

$$L' = \frac{\mu_0\left(ln\,\chi+\frac{1}{4}\right)}{\pi}. \quad (43)$$

So, the *characteristic impedance of Lecher line*, according to Eq. (39), Eq. (42) and Eq. (43), is:

$$Z_{\rm LC} = \frac{1}{2\,\pi\, f\, C} = \sqrt{\frac{L}{C}} = \sqrt{\frac{L'\,d\zeta}{C'\,d\zeta}} = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\sqrt{\left[ln\left(\frac{\chi}{2}+\sqrt{\left(\frac{\chi}{2}\right)^2-1}\right)\right]\left(ln\,\chi+\frac{1}{4}\right)}}{\pi} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\sigma(\chi)}{\pi}\ , \quad (44)$$

while

$$\sigma(\chi) = \sqrt{\left[ln\left(\frac{\chi}{2}+\sqrt{\left(\frac{\chi}{2}\right)^2-1}\right)\right]\left(ln\,\chi+\frac{1}{4}\right)} \quad (45)$$

we call the *structural coefficient the Lecher line*.

If we now express the frequency f from equations (35), (39), (44) and (45) we get:

$$f = \frac{1}{2\,\pi\, Z_{LC}\, C} = \frac{1}{2\,\pi\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\sigma(\chi)}{\pi}C} = \frac{1}{2\,\pi\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\sigma(\chi)}{\pi}4\pi\varepsilon_0 r} = \frac{1}{8\pi\sqrt{\varepsilon_0\mu_0}\,\sigma(\chi) r}. \quad (46)$$

If we now introduce r from equation (30) into equation (46), we get:

$$f = \frac{1}{\sqrt{\varepsilon_0\mu_0}\,\sigma(\chi)\frac{qQ}{\varepsilon_0 E_{\rm em}}\frac{1-\frac{E_{\rm em}}{mc^2}}{1-\frac{E_{\rm em}}{2mc^2}}} = \frac{E_{\rm em}}{\sqrt{\frac{\mu_0}{\varepsilon_0}}\sigma(\chi)\,zZe^2}\frac{1-\frac{E_{\rm em}}{2mc^2}}{1-\frac{E_{\rm em}}{mc^2}}. \quad (47)$$

Photon frequency (or wavelength, $f = c/\lambda$) information can be obtained from NIST via the Ritz wavelength [14].

4. Structural Constant of All Atoms

The characteristics of an LC oscillator are an inherent property of the oscillator itself and depend only on the structural parameters of the LC circuit under consideration, and do not depend on variables in that LC circuit, such as charges, currents or voltages in that LC circuit. The frequency given by equation (47) is also an inherent property of the LC oscillator and does not depend on the charge $\boldsymbol{qQ}$ appearing in that equation. In order to avoid this dependence of the natural frequency f on the charge product $\boldsymbol{qQ}$ in the LC circuit, in Eq. (47), the product $\boldsymbol{\sigma(\chi)\,qQ} = -\boldsymbol{\sigma(\chi)zZe^2} = \textbf{Constant 1}, \textbf{actually}, \textbf{since}\ \boldsymbol{e^2}$ is constant in itself, so it should be a constant $-\boldsymbol{\sigma(\chi)zZ} = \textbf{Constant 2} = \boldsymbol{s_0^2}$, with, as stated above, $q=z(-e)=-ze$ and $Q=Z(+e)=Ze$, where elementary charge e is used only as measure of the charge, but not as a variable; it should be kept in mind that at the beginning we said that the charge q

would be considered negative, *i.e.*, $q = -ze$, so $\sqrt{-\mathbf{1}} = \mathbf{i}$, and

$$\boldsymbol{s_0} = \left|\sqrt{\boldsymbol{\sigma(\chi)\,(zZ)}}\right| = \textbf{Constant 3}, \tag{48}$$

the factors z and Z theoretically do not necessarily have to be an integer. The larger z or Z, the proportionally smaller σ(χ), so that s_0 is kept constant. The emitted (absorbed) electromagnetic energy E_{em} depends on the variable charge Θ^2. However, as noted after equation (40), this does not affect the frequency *f*. Therefore, the energy E_{em} does not have to satisfy the stated requirement that the frequency does not depend on the variables and can remain as such in equation (47). The constant s_0 in Eq. (48) is the so-called the structure constant of all atoms (note here that it is not the fine structure constant α (we will show later that there is a strong connection between them, but they are two different physical constants).

If we now return to Eq. (41), using Eq. (47) and Eq. (48), and take $\sqrt{\frac{\mu_0}{\varepsilon_0}} = \mu_0 c$, we get:

$$A = \frac{E_{\text{em}}}{f} = \frac{E_{\text{em}}}{\frac{E_{\text{em}}}{\sqrt{\frac{\mu_0}{\varepsilon_0}}\,\sigma(\chi)\,zZe^2}\,\frac{1-\frac{E_{\text{em}}}{2mc^2}}{1-\frac{E_{\text{em}}}{mc^2}}} = \frac{1}{\frac{1}{\mu_0 ce^2 s_0^2}\,\frac{1-\frac{E_{\text{em}}}{2mc^2}}{1-\frac{E_{\text{em}}}{mc^2}}} = \mu_0 ce^2 s_0^2\,\frac{1-\frac{E_{\text{em}}}{mc^2}}{1-\frac{E_{\text{em}}}{2mc^2}} = A_0\,\frac{1-\frac{E_{\text{em}}}{mc^2}}{1-\frac{E_{\text{em}}}{2mc^2}}. \tag{49}$$

Here A_0 is the part of action of *LC* oscillator that does not depend on the speed of the electron motion, and this part is:

$$A_0 = \mu_0 ce^2 s_0^2. \tag{50}$$

According to Eq. (49) it is valid:

$$E_{\text{em}} = A_0\, f\,\frac{1-\frac{E_{\text{em}}}{mc^2}}{1-\frac{E_{\text{em}}}{2mc^2}}, \tag{51}$$

$$E_{\text{em}} = A\, f, \tag{52}$$

and from Eq. (49) is possible two solutions, after solving this equation with E_{em} as a variable:

$$E_{\text{em1}} = A_0 f + mc^2 - \sqrt{(A_0 f)^2 + (mc^2)^2}, \tag{53a}$$

$$E_{\text{em2}} = A_0 f + mc^2 + \sqrt{(A_0 f)^2 + (mc^2)^2} \tag{53b}$$

and since Eq. (52) and Eq. (53a), (53b)

$$A = A_0 + \frac{mc^2}{f} \mp \sqrt{A_0^2 + \left(\frac{mc^2}{f}\right)^2}. \tag{54}$$

This theoretically derived Planck's *h*. Namely, by definition, *h* is the ratio between the energy of a photon and its frequency, from Eq. (52); $A = h = E_{\text{em}}/f$, and whether that will be a constant or not, we will only see later after the measurements. In any case, in the theory presented here, the ratio of radiated energy and its frequency are not predicted in advance as either variable or constant, but are completely independent, and only their calculation or their measurements show what the real relationship between them is.

To check accuracy of Eq. (53a), (53b) and Eq. (54), it is necessary to determine the amount of the energy E_{em}, or ionization voltage V_{em}, the amount of the frequency *f*, and the amount of the structural constant s_0. All other quantities needed for the aforementioned verification of Eq. (53a), (53b) and Eq. (54) are already known ($\mu_0,\ c,\ e,\ m$). Starting from equation Eq. (29) and Eq. (51) we get Duane-Hunt law [8] with *relativistic correction* (originally $f = \frac{eV_{em}}{h}$, here *h* is Planck's constant), and reads:

$$f = \frac{eV_{\text{em}}}{A_0}\,\frac{1-\frac{eV_{\text{em}}}{2mc^2}}{1-\frac{eV_{\text{em}}}{mc^2}} = \frac{eV_{\text{em}}}{\mu_0 ce^2 s_0^2}\,\frac{1-\frac{eV_{\text{em}}}{2mc^2}}{1-\frac{eV_{\text{em}}}{mc^2}} = \frac{mc^2}{\mu_0 ce^2 s_0^2}\,\frac{(\frac{1}{2}\beta^2)}{\sqrt{1-\beta^2}} = \frac{mc^2}{2\mu_0 ce^2 s_0^2}\,\frac{\beta^2}{\sqrt{1-\beta^2}} = \frac{mc}{2\mu_0 e^2 s_0^2}\,\frac{\beta^2}{\sqrt{1-\beta^2}} = f_0\,\frac{\beta^2}{\sqrt{1-\beta^2}}. \tag{55}$$

Here $f_0 = \frac{mc}{2\mu_0 e^2 s_0^2}$ is the *natural frequency of the electron*, which belongs to the electron regardless of its speed of motion. It is also possible to calculate the natural frequencies for all other particles; *i.e.*, for protons, neutrons, hyperons, ... depending on their masses. An interesting expression is obtained if we multiply this expression for the natural frequency f_0, with the expression A_0, Eq. (50), we get: $A_0 f_0 = \mu_0 ce^2 s_0^2\,\frac{mc}{2\mu_0 e^2 s_0^2}\,\mu_0 ce^2 s_0^2 = \frac{1}{2}mc^2$, we get the classic expression for the kinetic energy of an electron that would move at the speed of light. The physical interpretation of this expression is not clear and we will not go into it now.

5. Discretization of the States in the Atom

In the atom there are at least two independent physical phenomena that enable the existence of the atom itself. One is the uniform circular motion of the electron around the nucleus with a speed *v* at a distance *r*, and the other is the oscillation of the electromagnetic energy generated within the atom. The time T_ϕ of one complete revolution of the electron around the nucleus (the so-called *period*) is:

$$T_\phi = \frac{2r\pi}{v}, \tag{56}$$

and from that

$$\phi = \frac{1}{T_\phi} = \frac{v}{2r\pi}, \tag{57}$$

ϕ is the *frequency of the rotation* of body charged with the charge q. Entirely different oscillation period, is period of *electromagnetic oscillation*, T_{em}, with frequency f:

$$T_{em} = \frac{1}{f}, \tag{58}$$

and

$$f = \frac{1}{T_{em}}. \tag{59}$$

Using Eq. (55), ($f = \frac{mc^2}{\mu_0 c e^2 s_0^2} \frac{(\frac{1}{2}\beta^2)}{\sqrt{1-\beta^2}}$), and Eq. (57), ($\phi = \frac{1}{T_\phi} = \frac{v}{\frac{-2qQ}{4\pi\varepsilon_0 mc^2}\frac{\sqrt{1-\beta^2}}{\beta^2}\pi}$), let's make a frequency ratio $\frac{f}{\phi}$:

$$\frac{f}{\phi} = \frac{\frac{mc^2}{\mu_0 c e^2 s_0^2}\frac{(\frac{1}{2}\beta^2)}{\sqrt{1-\beta^2}})}{\frac{v}{\frac{-2qQ}{4\pi\varepsilon_0 mc^2}\frac{\sqrt{1-\beta^2}}{\beta^2}\pi}} = -\frac{qQc}{4ve^2 s_0^2} \tag{60}$$

Electromagnetic energy in the atom can exist as a standing wave. The standing wave does not transmit the energy, but it sways existing energy. If the natural frequency of the *LC* oscillator is *f*, as the *active power* then standing wave oscillates with dual frequency $2f$ [9] (p.437-8),

$$f_{Sw} = 2f. \tag{61}$$

Thesis: *In order for the electromagnetic standing wave to exist in the atom, there must be a mutual synchronization relationship between the frequency of electron motion around the nucleus ϕ and the frequency f of oscillation of electromagnetic energy in the atom f* (it should be noted here that other integer relations between these two phenomena are theoretically possible):

$$\phi = n\, f, \tag{62}$$

where n is one of the whole numbers 1, 2, 3, …. Both above mentioned phenomena in respect of synchronization are equal; so also applies

$$f = n\, \phi. \tag{63}$$

The two Eqs. (62, 63), can be written as one expression

$$f = n^{\pm 1}\phi, \tag{64}$$

or, because of Eq. (61) and Eq. (64):

$$f = \frac{1}{2} f_{Sw} = \frac{1}{2} n^{\pm 1}\phi, \tag{65}$$

From equation (60) we obtain the equation for the particle velocity in the n-th state v_n:

$$v_n = -\frac{qQc}{4(\frac{f}{\phi})e^2 s_0^2}, \tag{66}$$

and due to equation (65) we get:

$$v_n = -\frac{qQc}{4(\frac{1}{2}n^{\pm 1})e^2 s_0^2} = -\frac{c}{2n^{\pm 1}s_0^2}\frac{qQ}{e^2} = -\frac{c}{2n^{\pm 1}s_0^2}\frac{(-ze)(+Ze)}{e^2} = \frac{czZ}{2n^{\pm 1}s_0^2} \tag{67}$$

According to Eq. (65), the frequency of the electromagnetic wave in the n-the state f_n is:

$$f_n = \frac{1}{2} n^{\pm 1}\phi_n, \tag{68}$$

From Eq. (67), taking into account $v_n = c\beta_n$, we obtain:

$$\beta_n = \frac{zZ}{2n^{\pm 1}s_0^2}. \tag{69}$$

Now we can express β_n from equation (29)

$$\beta_n = \sqrt{-\frac{2qV_{em}}{mc^2}\left(1+\frac{qV_{em}}{2mc^2}\right)} = \sqrt{-\frac{qV_{em}}{mc^2}\left(2+\frac{qV_{em}}{mc^2}\right)} \tag{70}$$

and equate it with β_n from equation (69). We get:

$$\sqrt{-\frac{qV_{\mathrm{em}}}{mc^2}\left(2+\frac{qV_{\mathrm{em}}}{mc^2}\right)}=\frac{zZ}{2n^{\pm1}s_0^2}\ . \tag{71}$$

From Eq. (71) we get structural constant s_0 determined by the *new method*, which proves to be the most accurate:

$$s_0=\frac{\sqrt{zZ}}{\sqrt{2n^{\pm1}\sqrt{1-\left(1-\frac{eV_{\mathrm{em(n)}}}{mc^2}\right)^2}}} \tag{72}$$

From equation (72) we obtain two solutions for $V_{\mathrm{em(n)}}$:

$$V_{\mathrm{em1(n)}}=\frac{mc^2}{ze}\left(1-\sqrt{1-\left(\frac{1}{n^{\pm1}}\frac{zZ}{2s_0^2}\right)^2}\right), \tag{73}$$

and

$$V_{\mathrm{em2(n)}}=\frac{mc^2}{ze}\left(1+\sqrt{1-\left(\frac{1}{n^{\pm1}}\frac{zZ}{2s_0^2}\right)^2}\right). \tag{74}$$

The solution in Eq. (73) is for the lower (ionization) voltages (namely, we will see this later after we determine s_0, for q=-e, z=1, $n^{\pm1}=1$) when Z is going from 0 to 137.0734789184, than ionization voltage $V_{\mathrm{em1(n)}}$ is going from 0 V to 510999 V, and the solution in Eq. (74), (q=-e, z=1, $Z=1/n^{\pm1}$) (See Fig. 2), when Z is going from zero (0) to 137.0734789184 and (ionization) voltages $V_{\mathrm{em2(n)}}$ goes in the opposite direction from 1 020 130 V to 510 99 V. Let us introduce now one abbreviation. Namely, instead of β, in accordance with Eq. (55), we put (z Z)/ ($n^{\pm1}2s_0^2$), so we get:

$$\Gamma_{\mathrm{n}}=\sqrt{1-\beta_{\mathrm{n}}^2}=\sqrt{1-\left(\frac{1}{n^{\pm1}}\frac{zZ}{2s_0^2}\right)^2}, \tag{75}$$

the symbol of the field of Discrete Physics.

6. Numerical Value of the Structural Constant

Derived formulas do not allow direct theoretical calculation of structural constant s_0. So, we have to do at least one precise measurement. This can be best done through Eq. (72), thanks to the precision of NIST data [6], [13] [https://www.nist.gov/pml/atomic-spectra-database; $V_{\mathrm{em(1)}}=13.5984345997\,V$, $n^{\pm1}=1$, $z=1$,

$Z=1$, q =-1.602176621×10^{-19} A·s, m=9.109383560×10^{-31} kg, c=299 792 458 m/s:

$$s_0=\frac{\sqrt{zZ}}{\sqrt{2n^{\pm1}\sqrt{1-\left(1+\frac{qV_{\mathrm{em(1)}}}{mc^2}\right)^2}}}=\frac{\sqrt{1\cdot1}}{\sqrt{2\cdot1\cdot\sqrt{1-\left(1-\frac{eV_{\mathrm{em(1)}}}{mc^2}\right)^2}}}=8.278\,691\,893\,077\,290. \tag{76}$$

The value of structural constant of atoms s_0 is the same for all atoms (Table 1). This is a Fundamental Physical Constant, (perhaps even in the category "Universal Constants", next to *characteristic impedance of vacuum*, *electric constant*, *magnetic constant*, *Newtonian constant of gravitation*, *Planck constant*, or *speed of light in vacuum*). When we know s_0 then we can from Eqs. (73, 74) calculate $V_{\mathrm{em1}(n)}$, $V_{\mathrm{em2}(n)}$, and all other sizes (Figure 2).

Table 1. Structural constant of atoms s_0 calculated on the basis of ionization voltage according to NIST's data*

Chemical symbol of the element	Atomic number	Ionization voltage, V_{em1}	Structural constant s_0
	Z Eq. (74), n^0	V_{em1}, [Volt]	[Dimensionless Number], Eq. (72)
n^0, H	1 (712207.805 V) [a]	13.598 434 49 [b] ←→	8.278 691 910 036
He	2	54.417 7650 [b] ←→	8.277 860 595 602
Li	3	122.454 3581 [b] ←→	8.277 755 226 469
Be	4	217.718 5843 [b] ←→	8.277 739 533 105
B	5	340.226 020 [b] ←→	8.277 739 896 484

Table 1. Continued

Chemical symbol of the element	Atomic number Z Eq. (74), n^0	Ionization voltage, V_{em1} V_{em1}, [Volt]		Structural constant s_0 [Dimensionless Number], Eq. (72)
C	6	489.993 194 [b]	↔	8.277 757 231 963
N	7	667.046 116 [b]	↔	8.277 771 755 296
O	8	871.409 88 [b]	↔	8.277 791 688 375
F	9	1 103.117 47 [b]	↔	8.277 812 276 144
Ne	10	1 362.199 15 [b]	↔	8.277 842 618 038
Ca	20	5 469.8615 [b]	↔	8.278 203 152 859
Zn	30	12 388.929 [b]	↔	8.278 637 976 575
Zr	40	22 236.677 [b]	↔	8.279 106 265 650
Sn	50	35 192.39 [b]	↔	8.279 610 860 584
Nd	60	51 515.58 [b]	↔	8.280 166 600 276
Yb	70	71 574.80 [b]	↔	8.280 846 679 237
Hg	80	95 897.70 [b]	↔	8.281 775 555 833
Th	90	125 253.40 [b]	↔	8.283 267 729 872
Fm	100	160 804.00 [b]	↔	8.286 011 987 216
Ds	110	204 394.00 [b]	↔	8.291 558 770 012
Rg	111	211 181.96 [c]	←	8.278 691 910 036 [d]
Cn	112	216 395.64 [c]	←	8.278 691 910 036 [d]
Fl	114	227 257.00 [c]	←	8.278 691 910 036 [d]
Lv	116	238 755.14 [c]	←	8.278 691 910 036 [d]
Og	118	250 947.50 [c]	←	8.278 691 910 036 [d]
xx [e]	119	257 386.97 [c]	←	8.278 691 910 036 [d]
xx [e]	120	264 022.11 [c]	←	8.278 691 910 036 [d]
xx [e]	122	278 037.10 [c]	←	8.278 691 910 036 [d]
xx [e]	126	309 790.06 [c]	←	8.278 691 910 036 [d]
xx [e]	130	348 967.80 [c]	←	8.278 691 910 036 [d]
xx [e]	132	373 260.57 [c]	←	8.278 691 910 036 [d]
xx [e]	134	403 395.73 [c]	←	8.278 691 910 036 [d]
xx [e]	136	447 172.13 [c]	←	8.278 691 910 036 [d]
xx [e]	137	494 269.44 [c]	←	8.278 691 910 036 [d]

*NIST: National Institute of Standards and Technology. https://en.wikipedia.org/wiki/National_Institute_of_Standards_and_Technology

[a] Neutron, according to the assumption herein, it is treated as hydrogen atom in the discrete state n^{-1}=126, according to Eq. (74), V_{em2}

[b] Data is from NIST Atomic Spectra Database Ionization Energies Form; https://physics.nist.gov/PhysRefData/ASD/ionEnergy.html

[c] No data are available by NIST (June 30, 2020); the amount of V_{em1} obtained by using Eq. (73) with s_0=8.278 691 910 036.

[d] Structural constant of hydrogen is used here; s_0=8.278 691 910 036, as the most accurately measured amount, for calculating V_{em1} according to Eq. (73).

[e] Element has not been revealed yet, but in accordance with here presented theory the highest atomic number can be Z=137.

Let us now return to Eq (30) and Eq. (33) with new insights. Substituting the two equations (55) and (61) into equation (16), using q=-ze, Q=Ze, $\varepsilon_0 = 1/(\mu_0 e^2$ we obtain:

$$r_\mathrm{n} = \left| \frac{\mu_0 c^2 Ze}{8\pi V_\mathrm{em}} \frac{1-\frac{zeV_\mathrm{em}}{mc^2}}{1-\frac{zeV_\mathrm{em}}{2mc^2}} \right| = (n^{\pm 1})^2 \frac{\mu_0 e^2 s_0^4}{\pi mzZ} \Gamma_\mathrm{n} = (n^{\pm 1})^2 \frac{\mu_0 e^2 s_0^4}{\pi mzZ} \sqrt{1-\left(\frac{1}{n^{\pm 1}} \frac{zZ}{2s_0^2}\right)^2}. \quad (77)$$

So, for example, repeat the calculation as we did after Eq. (33), but now in a different way. There we should measure V_em and used 6 more parameters; *i.e.*, *z*, *Z*, *e*, μ_0, *m*, *c*, while now, instead of measuring V_em we have new parameters s_0 and $n^{\pm 1}$, and we need 7 additional parameters; *z*, *Z*, *e*, μ_0, *m*, *n*, s_0, and we don't need the speed of light *c*. The results are the same as before, *i.e.*, for the first orbit of Hydrogen (z=1, Z=1, e=1.602 176 621×10^{-19}A·s, μ_0=4π×10^{-7} kg·m·A^{-2}·s^{-2}, m=9.109 383 560×10^{-31} kg, $n^{\pm 1}$=1, s_0=8.278 691 893 077 290) r_H = 5.294 526 279×10^{-11} m, and we get the radius of the first orbit of Helium (z=1, Z=2, $n^{\pm 1}$=1) r_He = 2.647 051 780×10^{-11} m. Also, we get the first orbit of Neon (z=1, Z=10, $n^{\pm 1}$=1) r_Ne = 5.280 558 679×10^{-12} m, while for Darmstadtium the first orbit is (z=1, Z=110, $n^{\pm 1}$=1) r_Ds = 2.871 955 104×10^{-13} m. So, by introduction structural constant s_0 we avoided the measurement of V_em and got all the same results as before. The resulting deviations are most commonly inside ±0.1% and at Darmstadtium (Z=110) reach a maximum of (measured 204394 V- calculated 206103 V)/ (204394 V) = -0.84%, calculated with s_0=8.278691910036, and the deviation would fall to zero with s_0=8.291604406878514, means with a decrease in s_0 of 0.16%. These differences should be attributed to the accuracy of the measurements and subsequently corrected.

Now we can also calculate the *natural frequency of the electron*, according to Eq. (55) $f_0 = \frac{mc}{2\mu_0 e^2 s_0^2} = 6.176\,260\,617\,278 \cdot 10^{19}$ Hz, of the proton, 1.134 055 759 542 · 10^{23} Hz, of the neutron, 1.135 618 966 399 · 10^{23} Hz.

Let us now calculate the frequency f_n of the discrete system using equations (29), (53a) and (75). From Eq. (29) get: $E_\mathrm{em} = mc^2\left(1-\sqrt{1-\beta^2}\right)$, from Eq. (53*a*) we get $E_\mathrm{em} = A_0 f + mc^2 - \sqrt{(A_0 f)^2 + (mc^2)^2}$, while Eq. (75) gives us $\sqrt{1-\beta^2} = \Gamma_\mathrm{n} = \sqrt{1-\left(\frac{1}{n^{\pm 1}}\frac{zZ}{2s_0^2}\right)^2}$. So, we get:

$$E_{\mathrm{em}(n)} = A_0 f_\mathrm{n} + mc^2 - \sqrt{(A_0 f_\mathrm{n})^2 + (mc^2)^2}. \quad (78)$$

Now using equations (29) and (75) we get:

$$mc^2(1-\Gamma_\mathrm{n}) = A_0 f_\mathrm{n} + mc^2 - \sqrt{(A_0 f_\mathrm{n})^2 + (mc^2)^2}. \quad (79)$$

If we divide equation (79) by mc^2 and average, we get:

$$\Gamma_\mathrm{n} + \frac{A_0 f_\mathrm{n}}{mc^2} = \sqrt{\left(\frac{A_0 f_\mathrm{n}}{mc^2}\right)^2 + 1}. \quad (80)$$

After squaring Eq. (80) and arranging, we obtain the required frequency f_n:

$$f_\mathrm{n} = \frac{(1-\Gamma_\mathrm{n}^2)\, mc^2}{2\Gamma_\mathrm{n} A_0}. \quad (81)$$

Using equations (50) and (75) from Eq. (81) we finally obtain the expression for calculating the radiation (or absorption) frequency of an atom:

$$f_\mathrm{n} = \frac{mcz^2 Z^2}{8e^2\mu_0 (n^{\pm 1})^2 s_0^6 \sqrt{1-\left(\frac{1}{n^{\pm 1}}\frac{zZ}{2s_0^2}\right)^2}}. \quad (82)$$

Let us check expression (82) for several atoms.

1. H. For the hydrogen atom (z = 1, Z = 1, $n^{\pm 1}$ = 1) we get: $f_{\mathrm{n(H)}}$= 3.287231567560605×10^{15} Hz. According to equation (53) the energy of that photon is:

$E_\mathrm{em} = A_0 f + mc^2 - \sqrt{(A_0 f)^2 + (mc^2)^2}$ = 2.17870939977842 × 10^{-18} J, which divided by the electron charge *e* gives the ionization voltage $V_{\mathrm{em}(1)}$ 13.59843459966715 V, which corresponds to the known measured ionization voltage. Planck's *h*, the ratio of energy E_em to frequency *f*, is $h_{\mathrm{(H)}}$ = 6.62779410 × 10^{-34} J·s [Δ= (6.62607015-6.62779410)/ (6.62607015) = -0.11 %].

2. For He. Let's check the same for the other atoms. Frequency $f_{\mathrm{n(He)}}$ according to expression (87) for helium (z = 1, Z = 2, $n^{\pm 1}$ = 1) is: 1.3149976174765548 × 10^{16} Hz, and the electromagnetic energy is 8.715185508166792 ×10-

[18] J, which gives an ionization voltage of 54.395909876198296 V, which is only 0.04% higher than measured. Planck's *h*, is $h_{(\mathrm{He})}$ = 6.62752950 ×10^{-34} J·s [Δ= (6.6260715-6.62752950)/ (6.62607015) = - 0.022 %].

3. Ne. For neon (*z*=1, *Z*=10, $n^{\pm 1}$ = 1) we get:
$f_{\mathrm{n(Ne)}} = 3.29592662088256$ ×10^{16} Hz, which corresponds to the photon energy of 2.18158700065472124 ×10^{-17} J, that is, after dividing by the charge *e*, the ionization voltage of 1361.6395208573028 V, which is - 0.041 % less than measured. Planck's *h*, is $h_{(\mathrm{Ne})}$ = 6.61903995930 ×10^{-34} J·s [Δ= (6.62607015-6.62752950)/ (6.62607015) = - 0.11 %].

4. Ds. Finally, let's check everything the same as before for Darmstadtium, $_{110}$Ds, (*z* = 1, *Z* = 110, $n^{\pm 1}$ = 1). We get: $f_{\mathrm{n(Ds)}} = 6.6661095875955016$ ×10^{19} Hz. The energy according to Eq. (53a) is 3.3021338880755874 ×10^{-14} J, and the ionization voltage is 206 103 V, which is +0.83% higher than the measured voltage of 204 394 V. Planck's *h*, is $h_{(\mathrm{Ds})}$ = 4.95361476538 ×10^{-34} J·s [Δ= (6.62607015-4.95361476538)/ (6.62607015) = +25.24 %; this decline is visible in Figure 2].

In conclusion for this part, it can be said that formula (82), as a new formula for determining the frequency of photons, proves to be suitable with minimal deviations, and also that formula (53a, b) has proven its correctness in comparison with measurements.

Measurement is the most important proof of any theory. We will analyze an experiment conducted on school equipment from the LAYBOLD company in the field of X-ray (Röntgen) radiation. At low energies there is no difference between the official Planck's *h* and the measured Planck's *h* (see Figure 2). For this reason, we perform measurements in the Röntgen region of high energies. Although the accuracy of the measurements in this experiment is not great, the measurements will still result in two important conclusions that are relevant to this article.

***Z* (Ionization voltage V_{em} LEYBOLD Physics Leaflets P6.3.3.3 | Comparison with my theory of individual elements)**

file:///C:/Users/milan/Downloads/p6333_e-1.pdf ***f = c/ λ* Ionization energy $E_{\mathrm{em}}=V_{\mathrm{em}}\, e$**

Planck's *h*= E_{em}/f

m = 9.109383560 × 10^{-31} kg, *c* = 299792459 m/s, *z* = 1, *Z* = Atomic number, *e* = 1.602176621×10^{-19} A·s, μ_0 = 12.56637061×10^{-7} kg·m·A^{-2} s^{-2}, $n^{\pm1}$=1, s_0 = 8.278691910

(Measured (22 000 V, $\lambda_{\min}$ =55.70 ×10^{-12} m) **(*f* = 5.382270341×10^{18} Hz)** (3.524788566 ×10^{-15} J) (**6.548888002 ×10^{-34} J·s**) |
Zr = 40 (22 236.712 V), Calculated Eq. (82) f_n = 5.4987655 × 10^{18} Hz (3.562714009 ×10^{-15} J) (6.4791160930 ×10^{-34} J·s

(Measured (24 000 V, $\lambda_{\min}$ =50.80×10^{-12} m) **(*f* = 5.901426339×10^{18} Hz)** (3.845223890 ×10^{-15} J) (**6.515753429 ×10^{-34} J·s**) |
Mo = 42 (24 572.213 V), Calculated Eq. (82) f_n = 6.09151792 × 10^{18} Hz (3.936900062 ×10^{-15} J) (6.462921246 ×10^{-34} J·s)

(Measured (26 000 V, $\lambda_{\min}$ =46.90×10^{-12} m) **(*f* = 6.392163284×10^{18} Hz)** (4.165659215 ×10^{-15} J) (**6.516822291 ×10^{-34} J·s**) |
Tc = 43 (25 787.047 V), Calculated Eq. (82) f_n = 6.40104242 × 10^{18} Hz (4.131540383 ×10^{-15} J) (6.454480553 ×10^{-34} J·s)

Measured (28 000 V, $\lambda_{\min}$ =43.33×10^{-12} m) **(*f* = 6.918819709×10^{18} Hz)** (4.486094539 ×10^{-15} J) (**6.483901485 ×10^{-34} J·s**) |
Rh = 45 (28 312.031 V), Calculated Eq. (82) f_n = 7.04703908 × 10^{18} Hz (4.536087416 ×10^{-15} J) (6.436869958 ×10^{-34} J·s)

Measured (30 000 V, $\lambda_{\min}$ =40.40×10^{-12} m) **(*f* = 7.420605396×10^{18} Hz)** (4.806529863 ×10^{-15} J) (**6.477274569 ×10^{-34} J·s**) |
Pd = 46 (29 622.678 V), Calculated Eq. (82) f_n = 7.3837886484×10^{18} Hz (4.746076214 ×10^{-15} J) (6.427697813 ×10^{-34} J·s)

Measured (32 000 V, $\lambda_{\min}$ =37.50×10^{-12} m) **(*f* = 7.994465547×10^{18} Hz)** (5.126965187 ×10^{-15} J) (**6.413143139 ×10^{-34} J·s**) |
Cd = 48 (32.341.587 V), Calculated Eq. (82) f_n = 8.08552915 × 10^{18} Hz (5.181693458 ×10^{-15} J) (6.408601542 ×10^{-34} J·s)

Measured (34 000 V, $\lambda_{\min}$ =35.50×10^{-12} m) (*f* = 8.444857972×10^{18} Hz) (5.447400511 ×10^{-15} J) (**6.450553141 ×10^{-34} J·s**) |
In = 49 (33 750.404 V), Calculated Eq. (82) f_n = 8.45083495 × 10^{18} Hz (5.407410824 ×10^{-15} J) (6.398670493 ×10^{-34} J·s)

Measured [19] (35 000 V, $\lambda_{\min}$ =34.40×10^{-12} m) **(*f* = 8.714897035×10^{18} Hz)** (5.607618164 ×10^{-15} J) (**6.434520272 ×10^{-34} J·s**) |
Sn = 50 (35 192.501 V), Calculated Eq. (82) f_n = 8.82598577 × 10^{18} Hz (5.638460234 ×10^{-15} J) (6.388476456 ×10^{-34} J·s)

First, the above measurements, regardless of their accuracy, clearly show that as the ionization voltage increases, Planck's *h*, defined as the ratio of photon energy E_{em} to its frequency *f*, decreases. For ionization voltages from 22 kV to 35 kV Planck's *h* drops from 6.54888 to 6.4345×10^{-34} J·s, therefore by 1.74 %. Secondly, the calculation according to the theory presented here also shows the same trend of decreasing Planck's *h* with increasing ionization voltage, and this decrease in almost the same Röntgen range amounts from 6.4791 to 6.3884 × 10^{-34} J·s, *i.e*., by 1.40 %. These relationships are best seen in Figure 2 (h/A_0, dark green line). Since the ionization voltages of individual elements are fixed, it would be easier to compare measured and theoretical results by adjusting the variable (measured) voltages to these fixed voltages of individual elements. This is not an adjustment of the measurement results, but an objective adjustment of the given ionization voltages to those voltages that can and should be adjusted during measurement, so that the comparison of measured and calculated results would be as easy and accurate as possible [12]. The measurements in reference [13] were performed in a narrow range [from 7.98 kV, Cr=24 (7.89 kV)] to 8.86 kV, Mn = 25 (8.57 kV) and are not relevant for assessing the effect on Planck's *h* (see Fig. 2), namely, in that region Planck's *h* does not theoretically or practically change much, so that region is not interesting for us to research.

7. Is Planck's *h* Constant?

From the previous calculations we see that Planck's *h* is not constant. Let's take a closer look at why this is so. From expression (54), which by definition (https://www.britannica.com/science/Plancks-constant) should represent Planck's constant h, it can be seen that the quantity A=h decreases with increasing frequency, therefore Planck's *h* changes, that is, with increasing frequency, this expression decreases. Here lies the proof that Planck's *h* is not a constant. We are no longer talking about Planck's constant, but about Planck's h. Namely since A_0 is a constant ($A_0 = \mu_0 c e^2 s_0^2$) and all quantities in next Eq. (83)

$$A = h = A_0 + \frac{mc^2}{f} \mp \sqrt{A_0^2 + \left(\frac{mc^2}{f}\right)^2}, \tag{83}$$

($i.e.,\ \mu_0, c, e, s_0, m$) are also constants except frequency *f*, it is clear that with an increase in frequency *f*, the quantity *h* and *A* decreases (or increase) it falls eventually with increasing frequency to finally zero, A = 0, or $A = 2A_0$. While the physical meaning of A = 0 is clear, the physical meaning of $A = 2A_0$ has not yet been explored at this point. Therefore, we come to the conclusion that Planck's *h* is not a constant, as has been unconditionally believed for over a hundred years. Planck's *h* is a constant only for low frequencies *f*. The only constant for all frequencies *f*, as shown theoretically and by measurements on 110 types of atoms, is the structural constant of the atom s_0 =8.278 691 893 077 290. The verification was performed on all NIST data available so far (see Table 1). It will be interesting to check our calculated data for V_{em} in that table marked with ⟵, when data for new measurements above the chemical element Darmstadium ($_{110}$Ds) appear (14 *sizes in the lower part of this table, which NIST has not yet publicly presented, so we dared to calculate these quantities from our theoretical data*).

According to the NIST measurement, Planck's *h* does not change over time [11]. However, this does not contradict the theory presented here that Planck's *h* varies depending on ionization voltages V_{em} and photon frequency *f* [12].

Planck's *h* is also questioned by the theories of other authors [15], [16], [17], [18], [19], [20], [21].

The capacitance *C* can be derived from equations (21), $C = 4\pi\varepsilon_0 r$, and (68), $r_n = (n^{\pm 1})^2(\mu_0 e^2 s_0^4)\Gamma_n/(\pi m z Z)$. After inserting $\varepsilon_0 = 1/(\mu_0 c^2)$ and the expression for r_n into the equation for capacity, and rearranging, we get:

$$C_n = \frac{(2n^{\pm 1} e s_0^2)^2}{mc^2 zZ}\Gamma_n = \frac{(2n^{\pm 1} e s_0^2)^2}{mc^2 zZ}\sqrt{1 - \left(\frac{1}{n^{\pm 1}}\frac{zZ}{2s_0^2}\right)^2}. \tag{84}$$

For the first orbit of a hydrogen atom (n = 1, z = 1, Z = 1, $n^{\pm 1}$) it is worth: C_1=5.890 954 960×10^{-21} F.

The inductance *L* can be expressed from equation (36), $L = \frac{1}{\omega^2 C} = \frac{1}{4\pi^2 f^2 C}$, with the help of equations (82), and (84).

After substituting the above expression and arranging them, we get:

$$L_n = \frac{1}{\omega^2 C_n} = \frac{1}{4\pi^2 f^2 C_n} = \frac{(2n^{\pm 1}\mu_0 e s_0^4)^2}{m\,\pi^2 z^3 Z^3}\sqrt{1 - \left(\frac{1}{n^{\pm 1}}\frac{zZ}{2s_0^2}\right)^2}. \tag{85}$$

For the first orbit of a hydrogen atom (n = 1, z = 1, Z = 1, $n^{\pm 1}$) it is worth: L_1=3.9791835605×10^{-13} H.

The *characteristic impedance* of an *LC* circuit is, according to expression (39), equal to:

$$Z_{LC(n)} = \sqrt{\frac{L_n}{C_n}} = \sqrt{\frac{\frac{1}{(4\pi^2 f_n{}^2 C_n)}}{C_n}} = \sqrt{\frac{1}{4\pi^2 f_n{}^2 C_n{}^2}} = \frac{1}{2\,\pi\, f_n\, C_n} = \frac{c\mu_0 s_0^2}{\pi\, z\, Z}, \tag{86}$$

which, for the first hydrogen orbital, is Z_{LC} = 8218.719068068 ohms. Here, $c\mu_0$ is actually the wave resistance of the vacuum, equal to 376.7303 134 12 Ω. https://en.wikipedia.org/wiki/Impedance_of_free_space . From Eq. (86) it follows that the structural constant s_0 is equal to:

$$s_0 = \sqrt{\frac{\pi z Z}{c\mu_0}Z_{LC}} = 8.278\ 691\ 910, \tag{87}$$

which corresponds to the previously determined amount.

8. Application of Structural Constant s_0 - the Maximum Number Z in Periodic Table

The maximum number Z= Z_{max} that a type of atom can reach in the periodic table is determined by the electron velocity v_{max}, which can be reached by an electron (z=1) in its first shell ($n^{\pm 1} = 1$) relative to the speed of light, *i.e.*, when beta $ß_{max}$ =1. Therefore, the solution to this problem is found in the equation:

$$\beta_{max} = \frac{v_{max}}{c} = \frac{1}{n^{\pm 1}} \frac{zZ_{max}}{2s_0^2} = 1, \tag{88}$$

and that solution reads, the *maximum possible number of atoms* Z_{max}, according to these calculations, is $2s_0^2 =$ 137.0734789184. Accordingly, if the largest possible number of atoms is 137.0734789184, then the numerical distance between two neighboring atoms is 1/137.0734789184, which is 0.0072953573, and which is the *fine structure constant*, or a *unit of measurement* (boscovich, B) for a *new physical quantity*, a *type of substance*, whose introduction I personally advocate, *i.e.*, in addition to m, kg, s, A, K, cd, and mol, I propose adding B as the *eighth base unit of measurement* [10].

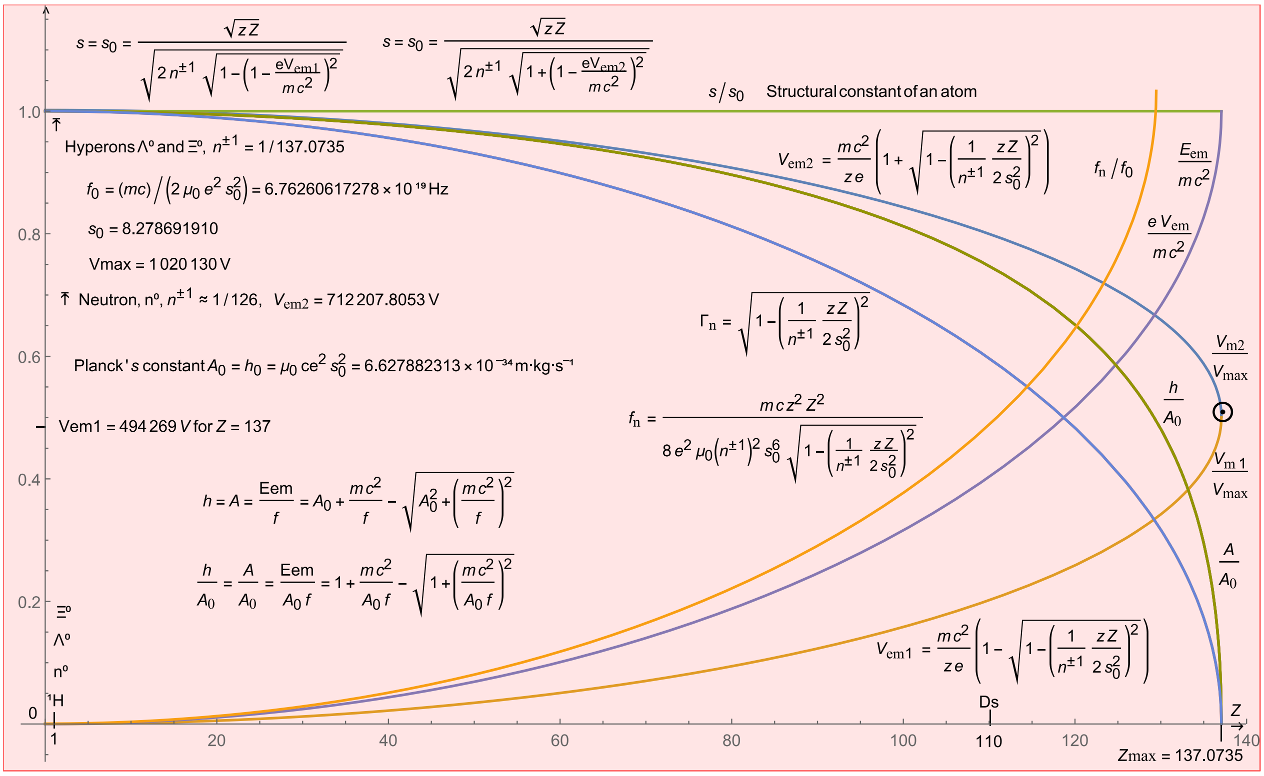

Figure 2. Photon electromagnetic energy E_{em}, and electron ionization energy eV_{em} from ionization voltage 0 (zero) V to ionization voltage V_{em1} = 494269 V, ionization voltage V_{em2} from ionization voltage 494269 V to ionization voltage V_{max}=1020130 V, photon frequency f_n, structural constant of an atom s, action of LC oscillator A and Γ_n - symbol of discrete physics, all this versus atomic number Z. Note that Planck's h is not a constant and that the only constant of all the quantities shown is the structural constant of an atom s=s_0=8.278691893077290, independent of the atomic number Z, as confirmed by NIST measurements on 110 atoms, from hydrogen ($_1$H) to Darmstadtium ($_{110}$Ds), as shown in Table 1. Therefore, the theoretically derived structural constant s_0, Eq. (72), confirmed by at least 110 NIST V_{em1} measurements.

9. Application of Structural Constant s_0- nine Interchangeable Constants

The use of the structural constant has many applications. Here we will highlight three such applications in particular. One application is that the structural constant makes 9 existing constants redundant (Table 2). Another application of the structural constant is the transfer of continuous theory to discrete theory or Discrete Physics (Table 3). The third application of the structural constant, on which the introduction of the unit for the type of substance is based, 'boscovich', B=1/($2s_0^2$), [10].

Table 2. Six (6) initial constants (s_0, c, μ_0, e, m, m_p) convert (9) nine below displayed constants in interchangeable

Quantity	Symbol	Formula	Value	Unit	Difference [a]
- Structural constant	s_0	s_0	8.278 691 910 [b]	1	unknown
- Speed of light in vacuum	c	c	299 792 458	$m \cdot s^{-1}$	0.0000
- Magnetic constant	μ_0	μ_0	$1.256\ 637\ 061 \times 10^{-6}$	$kg \cdot m \cdot A^{-2} \cdot s^{-2}$	0.0000
- Elementary charge	e	e	$1.602\ 176\ 621 \times 10^{-19}$	A·s	0.0000
- Electron mass	m	m	$9.109\ 383\ 560 \times 10^{-31}$	kg	0.0000
- Proton mass	m_p	m_p	$1.672\ 621\ 898 \times 10^{-27}$	kg	0.0000
	Down:	*9 interchangeable constants*			
1. Fine-structure constant	α	$1/(2s_0^2)$	$7.295\ 357\ 233 \times 10^{-3}$	1	-0.0273 [c]
1a) Inverse fine-str. cons.	α^{-1}	$2s_0^2$	$1.370\ 734\ 795 \times 10^{2}$	1	+0.0273 [c]
2. von Klitzing constant	R_K	$\mu_0 c s_0^2$	$2.581\ 986\ 745 \times 10^{4}$	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-2}$	+0.0273 [c]
3. Planck's constant	h	$\mu_0 c e^2 s_0^2$	$6.627\ 882\ 313 \times 10^{-34}$	$m \cdot kg \cdot s^{-1}$	+0.0273 [c]
3a). Conversion constant	K_0	$1/(2\mu_0 c e s_0^2)$	$1.208\ 664\ 053 \times 10^{14}$	$m^2 \cdot kg \cdot s^{-4} \cdot A^{-1}$	unknown
4. Ratio $e/h = 2K_0$	e/h	$1/(\mu_0 c e s_0^2)$	$2.417\ 328\ 106 \times 10^{14}$	$m^2 \cdot kg \cdot s^{-4} \cdot A^{-1}$	-0.0273 [c]
5. Josephson constant, $4K_0$	K_J	$2/(\mu_0 c e s_0^2)$	$4.834\ 656\ 212 \times 10^{14}$	$m^2 \cdot kg \cdot s^{-4} \cdot A^{-1}$	-0.0273 [c]
6. Rydberg constant	R_∞	$m/(8\mu_0 e^2 s_0^6)$	$1.096\ 473\ 231 \times 10^{7}$	m^{-1}	-0.0819 [c]
7. Bohr radius	a_0	$(\mu_0 e^2 s_0^4)/(\pi m)$	$5.294\ 667\ 174 \times 10^{-11}$	m	+0.0546 [c]
8. Bohr magneton	μ_B	$(\mu_0 c e^3 s_0^2)/(4\pi m)$	$9.276\ 546\ 489 \times 10^{-24}$	$A \cdot m^2$	+0.0273 [c]
9. Nuclear magneton	μ_N	$(\mu_0 c e^3 s_0^2)/(4\pi m_p)$	$5.052\ 165\ 117 \times 10^{-27}$	$A \cdot m^2$	+0.0273 [c]

[a] It is the difference with "2014 CODATA recommended values" in percent.

[b] *These calculation is based on the values provided by NIST, Eq. (76),* V_{em} =13.598 434 49 V, https://www.nist.gov/pml/atomic-spectra-database.

[c] This difference disappears completely if Planck's constant instead of h = 6.626 070 040 × 10^{-34} J·s is equal to $A_0 = \mu_0 c e^2 s_0^2$= 6.627 882 313 1934 × 10^{-34} J·s, *i.e.*, when it is increased by +0.0273%, with the note that A_0 here is a theoretically calculated value and confirmed by 110 NIST ionization voltage measurements. All discrepancies would disappear if the current value of Planck's h were increased by 0.0273%, as suggested by the expression $A_0 = \mu_0 c e^2 s_0^2$ = 6.627 882 313 1934 × 10^{-34} J·s.

10. Application of Structural Constant s_0 -Neutron as a Hydrogen Atom in State $n^{\pm 1}$= 1/126 and Explanation Why Is a Neutron +0.138% Heavier Than a Proton

If, for any reason, an electron falls below the first orbit (K-Electron Capture; Hideki Yukawa), then the atom in that state can still exist. Let us assume, then, that the neutron is actually a hydrogen atom in one of these states of his electron below the first orbit. Now we will determine what number $n^{\pm 1}$ (below the level $n^{\pm 1}$=1), belongs to that state of the hydrogen atom or the neutron. According to NIST, the mass of the proton is $m_P = 1.67262192369 \times 10^{-27} kg$, the mass of the neutron is $m_{n^0} = 1.67492749804 \times 10^{-27} kg$and the mass of the electron is $m = 9.1093837015 \times 10^{-31} kg$. The mass of hydrogen is the sum of mass of proton and the mass of electron in moving, that is, using Eq. (75) (for hydrogen z=1, Z=1, $n^{\pm 1} = 1$,), $m_H = m_P + m/\sqrt{1-\beta_H^2} = m_P + m/\Gamma_H = 1.67353295896 \times 10^{-27}$kg The difference between the mass of neutron and the mass of hydrogen is $\Delta m_H = m_{n^0} - m_H = 1.39453907917 \times 10^{-30} kg = (m_P + m/\sqrt{1-\beta_{n^0}^2}) \quad -(m_P + m/\sqrt{1-\beta_H^2}) \quad = m/\sqrt{1-\beta_{n^0}^2} - m/\sqrt{1-\beta_H^2} = m/\Gamma_{n^0} - m/\Gamma_H$, should then be attributed to the increase in the mass of electron in the observed hydrogen atom as neutrons. It follows from here $\Gamma_{n^0} = m\Gamma_H / (\Delta m_H \Gamma_H + m)$. According to Eq. (75), with $z = Z = n^{\pm 1} = 1$, $\Gamma_H = 0.999973389$. Therefore, the solution of the previous equation written in another form.

11. Application of Structural Constant s_0 -Link of Conversion Classical to Discrete Physics

Table 3. Link of Continuous (Classical) to Discrete Physics

Quantity	Symbol	Eq.	Continuously	Eq.	Discretely
Structural constant	s	Eq. (87)	$s=\sqrt{\pi z Z\sqrt{L/C}/(c\mu_0)}$	Eq. (87) and using Eq. (86)	s_0 = 8.278 691 910
Ratio $v/c=\beta$	β	Eq (70)	$\beta=\sqrt{2eV_{em}(1-\frac{eV_{em}}{2mc^2})/(mc^2)}=\sqrt{eV_{em}(2-\frac{eV_{em}}{mc^2})/(mc^2)}$	Eq. (71) ($v_n=\beta_n c$)	$\beta_n=\frac{zZ}{2n^{\pm1}s_0^2}$
Ratio $(v/c)^2=\beta^2$	β^2	Eq (29)	$\beta^2=\frac{2eV_{em}}{mc^2}\left(1-\frac{eV_{em}}{2mc^2}\right)=\frac{eV_{em}}{mc^2}\left(2-\frac{eV_{em}}{mc^2}\right)$	Eq. (71)	$\beta_n^2=(\frac{zZ}{2n^{\pm1}s_0^2})^2$
Abbreviation	Γ	Eq. (75)	$\Gamma=\sqrt{1-\beta^2}$	Eq. (75)	$\Gamma_n=\sqrt{1-\left(\frac{1}{n^{\pm1}}\frac{zZ}{2s_0^2}\right)^2}$
Relativistic momentum	p	Eq. (8)	$p=m\,v/\sqrt{1-\beta^2}$	Eq. (8) and using Eq. (75)	$p_n=m\,v_n/\sqrt{1-\left(\frac{1}{n^{\pm1}}\frac{zZ}{2s_0^2}\right)^2}$
Orbit radius	r	Eq. (11)	$zZe^2\sqrt{1-\beta^2}/(4\pi\varepsilon_0 mc^2\beta^2)$	Eq. (77)	$r_n=(n^{\pm1})^2\sqrt{1-\left(\frac{1}{n^{\pm1}}\frac{zZ}{2s_0^2}\right)^2}/(\pi mzZ)$
Kinetic energy	K	Eq. (12)	$K=\frac{mc^2}{\sqrt{1-\beta^2}}-mc^2$	Eq. (12) and using Eq. (75)	$K_n=mc^2[1/\sqrt{1-\left(\frac{1}{n^{\pm1}}\frac{zZ}{2s_0^2}\right)^2}-1]$
Potential energy	U	Eq. (13)	$U=-\frac{mc^2\beta^2}{\sqrt{1-\beta^2}}$	Eq. (31) and using Eq. (75)	$U_n=-\frac{mc^2(1-\Gamma_n^2)}{\Gamma_n}$
Electromagnetic energy	E_{em}	Eq. (29)	$E_{em}=mc^2(1-\sqrt{1-\beta^2})=qV_{em}$	Eq. (29) and using Eq. (75)	$E_{em(n)}=mc^2(1-\Gamma_n)$
Ionization voltage	V_{em}	Eq. (73, 74)	$V_{em}=mc^2(1\pm\sqrt{1-\beta^2})/q$	Eq. (73,74) and using Eq. (75)	$V_{em(n)}=mc^2(1\pm\Gamma_n)/(ze)$
Coulomb potential	V_U	Eq. (31)	$V_U=-\frac{mc^2\beta^2}{q\sqrt{1-\beta^2}}$	Eq. (31) and using Eq. (75)	$V_{U(n)}=\frac{mc^2(1-\Gamma_n^2)}{ze\Gamma_n}$
Capacitance	C	Eq. (35)	$C=q\,Q\sqrt{1-\beta^2}/(mc^2\beta^2)$	Eq. (85), and using (75)	$C_n=\frac{(2n^{\pm1}es_0^2)^2\Gamma_n}{mc^2zZ}$
Frequency	f	Eq. (73)	$f=\frac{mc\beta^2}{2\mu_0e^2s_0^2\sqrt{1-\beta^2}}$	Eq. (86), Eq. (75), Eq. (82)	$f_n=\frac{mc(1-\Gamma_n^2)}{2\mu_0e^2s_0^2\Gamma_n}=\frac{mcz^2Z^2}{8e^2\mu_0(n^{\pm1})^2s_0^6\Gamma_n}$
Inductance	L	Eq. (36)	$L=\frac{1}{4\pi^2f^2C}=\frac{1}{4\pi^2f^2C}$	Eq. (85) and using Eq. (75)	$L_n=\frac{(2n^{\pm1}\mu_0es_0^4)^2\Gamma_n}{m\pi^2z^3Z^3}$
Characteristic impedance	Z_{LC}	Eq. (39)	$Z_{LC}=\sqrt{L/C}=1/(2\pi fC)$	Eq.(86)	$Z_{LC(n)}=\frac{c\mu_0s_0^2}{\pi zZ}$
LC oscillator action	$A=h$	Eq. (49)	$A=\frac{Eem}{f}=A_0\frac{1-Eem/(mc^2)}{1-Eem/(2mc^2)}$ Planck's h is a variable	Eq. (54)	$A=A_0+\frac{mc^2}{f}\mp\sqrt{A_0^2+\left(\frac{mc^2}{f}\right)^2}$
Planck's h ($6.626070040\times10^{-34}$ J·s)		Eq. (50)	$A_0=\mu_0ce^2s_0^2$, Planck's constant $h_0=6.627\,882\,313\times10^{-34}$ J·s		+0.0273 %

Therefore, the solution of the previous equation written in another form,

$$\sqrt{1-\left(\frac{1}{n_{\mathrm{n}^0}^{\pm 1}}\frac{zZ}{2s_0^2}\right)^2}=\frac{\sqrt{1-\left(\frac{1}{n_{\mathrm{H}}^{\pm 1}}\frac{zZ}{2s_0^2}\right)^2}}{1+\frac{\Delta m_{\mathrm{H}}}{m}\sqrt{1-\left(\frac{1}{n_{\mathrm{H}}^{\pm 1}}\frac{zZ}{2s_0^2}\right)^2}},\quad (89)$$

by parameter $n_{n^0}^{\pm 1}$ gives the result $n_{\mathrm{n}^0}^{\pm 1}=n_{\mathrm{n}^0}^{-1}=125.92000300221$. According to Eqs. (62, 63) here must be an integer, so that the real solution is$n_{n^0}^{-1}=126$, where the error is 0.064%. The highest ionization potential of neutrons is $V_{\mathrm{em}_{(n^0)}}=712207.8053\mathrm{V}$ (See Table 1). This is in good agreement with the calculated neutron mass. Namely, this calculated neutron mass is +0.08% greater than the measured neutron mass. This calculation method can be a logical explanation for the +0.14% difference in mass between the neutron mass and the proton mass. This means that due to the motion of the electron around the proton in the neutron, the mass of the neutron increased by +0.14% compared to the mass of the proton.

12. Method

Theoretical methods of Maxwell's equations and Special Theory of Relativity were used. All results were experimentally verified based on measurements obtained from publicly published NIST data. For computational verification of theoretical and measured results and for graphical presentation, the program "Wolfram Mathematica 12.1" was used.

13. Conclusion

Maxwell's theory of electromagnetism together with the theory of relativity here give good results in describing most phenomena in the atom, such as the radiation of electromagnetic energy, the discretization of states in the atom, the determination of stationary orbits, the determination and calculation of the structural constant of the atoms s_0. These two theories allow the transfer of the continuous theory to the discrete theory of the atom. Discrete theory, in addition to describing the states in the electron shell, that is, in higher atomic orbits, allows entry into orbits that are theoretically present below the first atomic orbit, which leads to the atomic nuclei. Therefore, the theory presented here explains that in atoms, in addition to the discrete states n^{+1} = 1, 2, 3, discrete states n^{-1} = 1, 2, 3 … are also present. Therefore, for example, it is possible that in the discrete state n^{-1} = 126 a hydrogen atom acquires the properties of a neutron. To achieve this, the existence of an electromagnetic oscillator inside the atom is assumed. This oscillator is described using the Lecher transmission line. The Lecher line does not actually exist within an atom however, a mathematical model of that line is used, just as a mathematical model is used in space exploration without the actual presence of planets in that model. The discretization of the atomic state is introduced via the integer ratio of the natural frequency of electromagnetic oscillations in the atom and the frequency of electron rotation around the atomic nucleus. The experimental results are in accordance with the presented theory. A new physical constant, the so-called structural constant of the atoms s_0 = 8.278691910, is introduced, explained in detail and verified by measuring at least 110 samples. This constant, with the help of 5 fundamental constants (c, μ_0, e, m, m_p) helps to make at least the remaining 9 physical constants interchangeable (fine structure constant α, von Klitzing constant R_K, Planck h, e/h ratio, Josephson constant K_J, Rydberg constant R_∞, Bohr radius a_0, Bohr magnetron μ_B, nuclear magnetron μ_N). The structural constant s_0 allows for the continuation of other research; one of them is the unit for the type of substance. The presentation in this article is based on atoms with one electron. Atoms with more electrons, on the basis set here, should be investigated separately. Planck's h, defined as the ratio of the energy of a photon to its frequency, has been theoretically calculated and experimentally verified. It has been shown that Planck's h is not constant and can only be considered constant in cases where the velocity of the observed body is much less than the speed of light. At higher velocities of the body, Planck's h gradually decreases to zero. The only real constant is the structural constant s_0, which is independent of the atomic number in Mendeleev's periodic table of all atoms. It turns out that the maximum possible number of atoms that can be reached in nature is equal to $2s_0^2$=137.073. Knowing the new physical constant s_0 allows to express the remaining 9 fundamental constants as redundant (fine structure constant α, von Klitzing constant R_K, Planck's h, ratio e/h, Josephson constant K_J, Rydberg constant R_∞, Bohr radius a_0, Bohr magneton μ_B, and nuclear magneton μ_N) with the help of other 5 constants (c, μ_0, e, m, m_p). Accurate knowledge of s_0 allows for the exact determination of Planck's constant h_0, instead of the currently accepted value of 6.626 070 15 × 10^{-34} J·Hz-

[1] to $h_0 = \mu_0\, c\, e^2\, s_0^2$ = 6.627 882 313 × 10^{-34} J·Hz^{-1}, that is, its increase by +0.0273 %. This article is written in a logical, physically clear sequence, so that all concepts are clear, as they are clear in classical physics.

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