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Using a Crossing Method as an Alternative Approach for Teaching, Learning, and Solving Quadratic Equations

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Abstract

This paper introduces the crossing method as an alternative approach for teaching and solving quadratic equations, particularly in secondary schools. Based on this research, the traditional methods used by in-service mathematics teachers—factorization, the general formula (quadratic formula), the graphical method, and completing the square—all yield the same solutions despite differing in approach. However, the crossing method is not documented in existing literature nor commonly used in teaching, even though it produces the same accurate results as the traditional methods. By presenting this new approach and comparing its outcomes with those of the four common methods, this paper makes a significant contribution to the field of mathematics education. The implications of the crossing method in teaching and learning quadratic equations are substantial, as it may enhance conceptual understanding by providing an intuitive way for students to grasp quadratic equations, especially through visualization. Since the crossing method involves comparing intersections, it might also increase student engagement through an interactive learning process. Additionally, mathematics teachers and educators can incorporate this method as an alternative pedagogical tool to cater to different learning styles. This research suggests that the crossing method could be a valuable complement to existing methods, potentially making quadratic equations more accessible to students. By demonstrating that it yields correct and consistent solutions, this study establishes the method as a viable alternative in mathematics instruction.

Keywords

Mathematics teachers; Quadratic equations; Crossing method

1. Introduction

Developing mathematical knowledge and skills in secondary schools is essential, and quadratic equations are among the key topics taught in Tanzanian secondary schools. Solving quadratic equations is fundamental in mathematics across various disciplines (Tendere & Mutambara, 2020); however, studies indicate that many students struggle with learning quadratic equations, leading to poor performance (Harripersaud, 2021). A significant factor contributing to this challenge is that many pre-service and in-service teachers lack effective knowledge and techniques that facilitate students' understanding of the foundational concepts and procedures involved in solving quadratic equations (Kabar & Gozde, 2023). Several methods have traditionally been used in teaching and learning quadratic equations, including factoring, completing the square, the quadratic formula, and graphing (Bossé & Nandakumar, 2005; Harripersaud, 2021). However, each of these methods has limitations in helping students fully grasp the underlying concepts and solve problems efficiently. Conventional teaching approaches emphasize factoring, the quadratic formula, and completing the square, with teachers typically focusing on the standard form of quadratic equations (Kabar & Gozde, 2023). Studies further reveal that students exert considerable effort when solving quadratic equations using these methods (e.g., Clinch, 2018; Vaiyavutjamai, Ellerton & Clements, 2005), suggesting a need for alternative approaches that enhance conceptual understanding and problem-solving skills.

This paper introduces the crossing method as a new approach to teaching and learning quadratic equations in a simple and understandable way for Tanzanian ordinary-level secondary schools. Deogratias (2022) previously introduced the crossing method as an alternative approach for teaching, learning, and solving systems of two linear equations in Tanzanian secondary schools. Building on this foundation, this paper provides a detailed description of the crossing method in solving quadratic equations, accompanied by examples to enhance comprehension. The method is designed to be simple and explanatory, making it effective for both teaching and learning quadratic equations. Furthermore, the paper compares the results obtained using the crossing method with those derived from traditional methods, including factoring, completing the square, the quadratic formula, and graphing, to highlight its effectiveness and potential advantages.

2. Materials and Methods

This study is a desktop research initiative that explores the various approaches used in teaching and learning quadratic equations in Tanzania and beyond. It examines existing methods, evaluates their effectiveness, and identifies potential alternative approaches that can enhance students' understanding and performance in solving quadratic equations. The crossing method was developed through a review of existing methods and validated through small-scale classroom trials in three secondary schools with students and in-service teachers, as well as one university with in-service mathematics teachers.

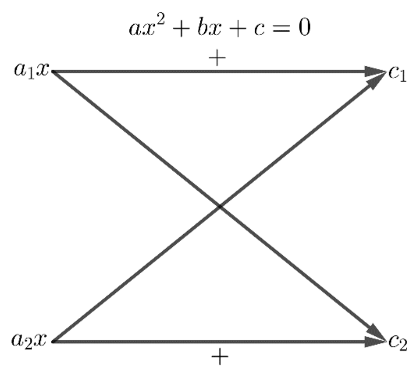
2.1 Introducing the crossing method as a new approach to teaching quadratic equations in Tanzania's ordinary level secondary schools

The crossing method is an alternative approach for teaching and learning quadratic equations that is simple, convenient, and time-efficient in finding correct solutions. It can be applied to quadratic equations with both positive and negative leading coefficients, making it a versatile technique. This method yields the same results as traditional methods such as factoring, completing the square, the quadratic formula, and the graphical method, as demonstrated in this paper, offering an effective and efficient alternative for solving quadratic equations.

2.2 Describing a Crossing method for teaching quadratic equations

Consider a quadratic equation $ax^2 + bx + c = 0$ and the following figure illustrating the crossing method for solving quadratic equations.

Crossing such an equation may be done as follows:



From the above figure, $(a_1x \times c_2) + (a_2x \times c_1) = bx$

Then $(a_1x + c_1)(a_2x + c_2) = 0$

Either $a_1x + c_1 = 0$ or $a_2x + c_2 = 0$

Thus, $a_1x = c_1$ or $a_2x = c_2$

Therefore $x = \frac{-c_1}{a_1}$ or $x = \frac{-c_2}{a_2}$

Note: In this case $a = a_1 \times a_2$ and $c = c_1 \times c_2$

3. Results

We present the results of the crossing method for solving quadratic equations by comparing them with the results obtained using other approaches, including the general formula for solving quadratic equations. This comparison highlights the effectiveness, accuracy, and efficiency of the crossing method as an alternative approach in teaching and learning quadratic equations.

3.1 Solving quadratic equations using the crossing method

Case I: When the Leading Coefficient is positive

Example: Solve the quadratic equation $12x^2 + 25x + 7=0$, by crossing method.

Solution

Given $12x^2 + 25x + 7 = 0$.

Here $a = 12, b = 25$ and $c = 7$

Let $a_1 = 4, a_2 = 3, c_1 = 7$ and $c_2 = 1$

Crossing:

$(4x \times 1) + (3x \times 7) = 25x$

Then, $(4x + 7)(3x + 1) = 0$.

This implies that $4x + 7 = 0$ or $3x + 1 = 0$

This implies that $4x = -7$ or $3x = -1$

Therefore, $x = \frac{-7}{4}$ or $x = \frac{-1}{3}$.

Case II: When the leading coefficient is negative

When teaching students to solve quadratic equations by using a crossing method when its leading coefficient is negative, we should consider that, one of the values of a should also be negative.

Example: Solve the quadratic equation $-x^2 - 2x + 3=0$, by crossing method.

Solution

Given $-x^2 - 2x + 3 = 0$

Here $a = -1, b = -2$ and $c = 3$

Let $a_1 = x, a_2 = -x, c_1 = -1$ and $c_2 = -3$

Crossing:

$(x \times -3) + (-x \times -1) = -2x$.

Then $(x + -1)(-x + -3) = 0$

Either $x + -1 = 0$ or $-x - 3 = 0$

Therefore $x = 1$ or $x = -3$

Case III: When the value of b is negative

When teaching students to solve quadratic equations by using a crossing method when the value of **b** is negative, we should consider that either one or both of the two values of **c** should also be negative.

Example: Solve the quadratic equation $x^2 - 5x + 6=0$ using the crossing method.

Solution

Given $x^2 - 5x + 6 = 0$

Here $a = 1, b = -5$ and $c = 6$

Let $a_1 = x, a_2 = x, c_1 = -2$ and $c_2 = -3$

Crossing:

$(x \times -3) + (x \times -2) = -5x.$

Then, $(x - 2)(x - 3) = 0.$

Either $x - 2 = 0$ or $x - 3 = 0$

Therefore, $x = 2$ or $x = 3.$

Case IV: When the value of c is negative

When teaching students to solve quadratic equations by using a crossing method when the value of b is negative, we should consider that one value of c should be negative.

Example: Solve the quadratic equation $6x^2 + x - 12 = 0$ using the crossing method.

Solution

Given $6x^2 + x - 12 = 0$

Here $a = 6, b = 1$ and $c = -12$

Let $a_1 = 2x, a_2 = 3x, c_1 = 3$ and $c_2 = -4$

Crossing:

$(2x \times -4) + (3x \times 3) = x$

Then $(2x + 3)(3x - 4) = 0.$

Either $2x + 3 = 0$ or $3x - 4 = 0$

Thus, $2x = -3$ or $3x = 4$

Therefore, $x = \frac{-3}{2}$ or $x = \frac{4}{3}$

3.2 Solving quadratic equations by using the general quadratic formula

Example: Solve the quadratic equation $12x^2 + 25x + 7=0$ using a general formula.

Solution:

Given $12x^2 + 25x + 7 = 0$

Here $a = 12, b = 25$ *and* $c = 7$

From the general quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-25 \pm \sqrt{25^2 - (4 \times 12 \times 7)}}{2 \times 12}$$

$$x = \frac{-25 \pm \sqrt{625 - 336}}{24}$$

$$x = \frac{-25 \pm \sqrt{289}}{24}$$

$$x = \frac{-25 \pm 17}{24}$$

$x = \frac{-25+17}{24}$ or $x = \frac{-25-17}{24}$

$x = \frac{-8}{24}$ or $x = \frac{-42}{24}$

Thus, $x = \frac{-1}{3}$ or $x = \frac{-7}{4}$

Therefore, $x = \frac{-1}{3}$ or $x = \frac{-7}{4}$

3.3 Solving quadratic equations by using factorization method

Example: Solve the quadratic equation $12x^2 + 25x + 7$=0 by using the factorization method.
Solution:

Given $12x^2 + 25x + 7 = 0$

Here $a = 12, b = 25$ and $c = 7$

$$12x^2 + 21x + 4x + 7 = 0$$

$$3x(4x + 7) + 1(4x + 7) = 0$$

$$(3x + 1)(4x + 7) = 0$$

Either $3x + 1 = 0$ or $4x + 7 = 0$

Thus, $3x = -1$ or $4x = -7$

Therefore, $x = \frac{-1}{3}$ or $x = \frac{-7}{4}$

3.4 Solving quadratic equations by completing the square method

Example: Solve the quadratic equation $12x^2 + 25x + 7$=0 by completing the square method.
Solution:

Given $12x^2 + 25x + 7 = 0$

Here $a = 12, b = 25$ and $c = 7$

$$x^2 + \frac{25}{12}x = \frac{-7}{12}$$

$$x^2 + \frac{25}{12}x + \frac{625}{576} = \frac{-7}{12} + \frac{625}{576}$$

$$\left(x + \frac{25}{24}\right)^2 = \frac{625 - 336}{576}$$

$$\left(x + \frac{25}{24}\right)^2 = \frac{289}{576}$$

Squaring both sides, we get

$$\left(x + \frac{25}{24}\right) = \sqrt{\frac{289}{576}}$$

$$\left(x + \frac{25}{24}\right) = \pm\frac{17}{24}$$

$$x = \frac{-25 \pm 17}{24}$$

$x = \frac{-8}{24}$ or $x = \frac{-42}{24}$

Thus, $x = \frac{-1}{3}$ or $x = \frac{-7}{4}$

Therefore, $x = \frac{-1}{3}$ or $x = \frac{-7}{4}$

3.5 Solving quadratic equation by graphing method

By drawing the graph of the quadratic equation $12x^2 + 25x + 7 = 0$. on the xy plane, we found that the graph cuts *x-axis* at two distinct points which are $x = \frac{-1}{3}$ and $x = \frac{-7}{4}$. This means $x = \frac{-1}{3}$ and $x = \frac{-7}{4}$ gives the solution of the quadratic equation.

4. Activities Performed by Teachers and Students on the Crossing Method

We described how to use the crossing method for solving, teaching, and learning quadratic equations to students and teachers. After that, we asked individual students and teachers to create and solve a quadratic equation using the crossing method. Below are some examples that students and teachers worked on during the meeting sessions.

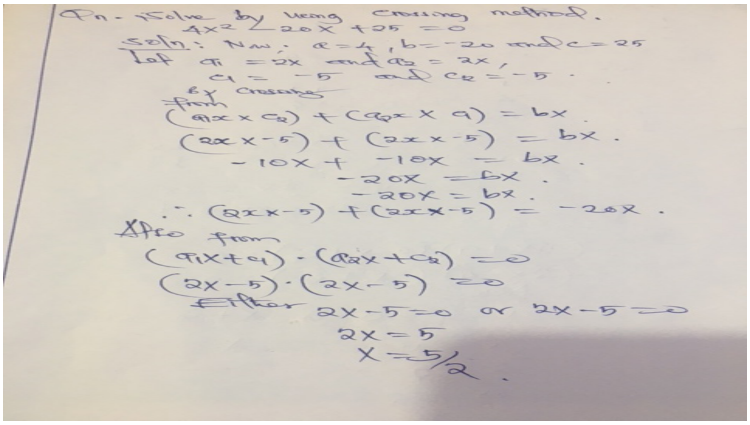

Qn. Solve by using crossing method.
$4x^2 - 20x + 25 = 0$
Soln: Now, $a = 4$, $b = -20$ and $c = 25$
let $a_1 = 2x$ and $a_2 = 2x$,
$c_1 = -5$ and $c_2 = -5$.
By crossing
from
$(a_1x \times c_2) + (a_2x \times c_1) = bx$.
$(2x \times -5) + (2x \times -5) = bx$.
$-10x + -10x = bx$.
$-20x = bx$.
$-20x = bx$.
$\therefore (2x \times -5) + (2x \times -5) = -20x$.
Also from
$(a_1x + c_1) \cdot (a_2x + c_2) = 0$
$(2x - 5) \cdot (2x - 5) = 0$
Either $2x - 5 = 0$ or $2x - 5 = 0$
$2x = 5$
$x = 5/2$.

Figure 1. Quadratic equation created and solved by Student 1 using the crossing method.

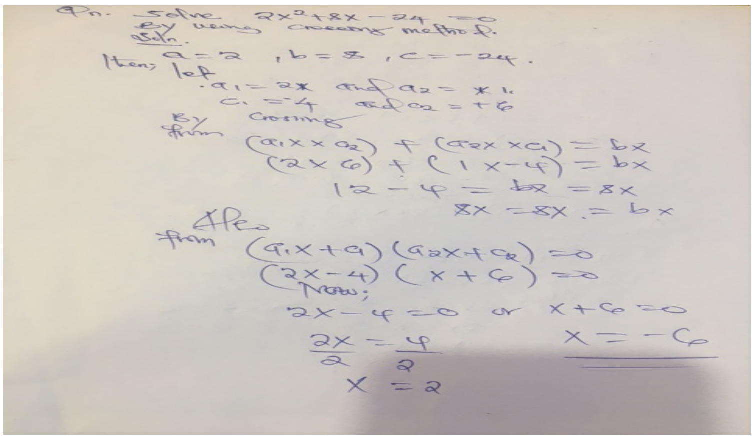

Qn. Solve $2x^2 + 8x - 24 = 0$
By using crossing method.
Soln.
Then, $a = 2$, $b = 8$, $c = -24$.
let $a_1 = 2x$ and $a_2 = x$.
$c_1 = -4$ and $c_2 = +6$
By crossing
from $(a_1x \times c_2) + (a_2x \times c_1) = bx$
$(2 \times 6) + (1 \times -4) = bx$
$12 - 4 = bx = 8x$
$8x = 8x = bx$
Also
from $(a_1x + c_1)(a_2x + c_2) = 0$
$(2x - 4)(x + 6) = 0$
Now;
$2x - 4 = 0$ or $x + 6 = 0$
$\frac{2x}{2} = \frac{4}{2}$ $\quad x = -6$
$x = 2$

Figure 2. Quadratic equation created and solved by Teacher 1 using the crossing method.

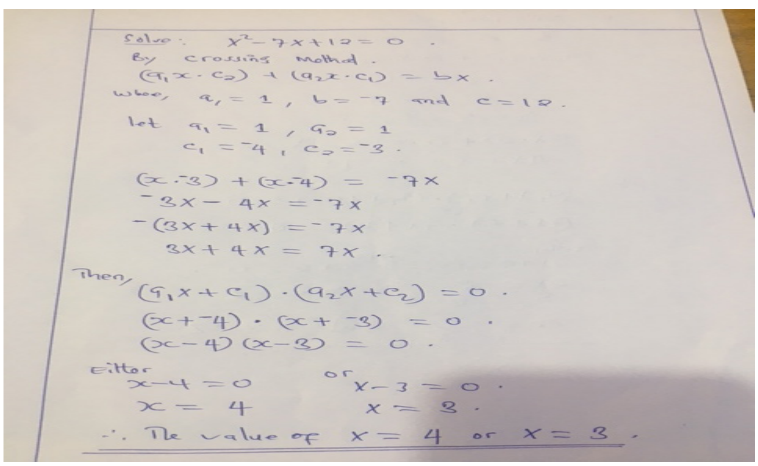

Solve: $x^2 - 7x + 12 = 0$.
By crossing method.
$(a_1x \cdot c_2) + (a_2x \cdot c_1) = bx$.
where, $a = 1$, $b = -7$ and $c = 12$.
let $a_1 = 1$, $a_2 = 1$
$c_1 = -4$, $c_2 = -3$.
$(x \cdot -3) + (x \cdot -4) = -7x$
$-3x - 4x = -7x$
$-(3x + 4x) = -7x$
$3x + 4x = 7x$.
Then, $(a_1x + c_1) \cdot (a_2x + c_2) = 0$.
$(x + -4) \cdot (x + -3) = 0$.
$(x - 4)(x - 3) = 0$.
Either $x - 4 = 0$ or $x - 3 = 0$.
$x = 4$ $\quad x = 3$.
$\therefore$ The value of $x = 4$ or $x = 3$.

Figure 3. Quadratic equation created and solved by Student 2 using the crossing method.

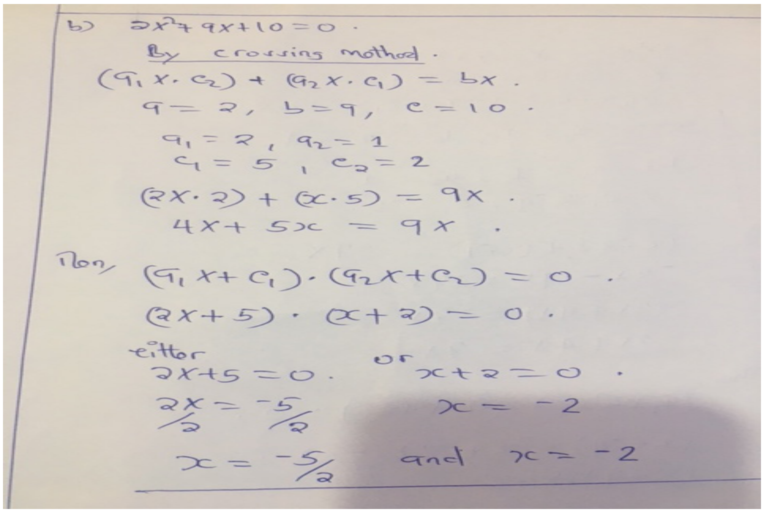


Figure 4. Quadratic equation created and solved by Teacher 2 using the crossing method.

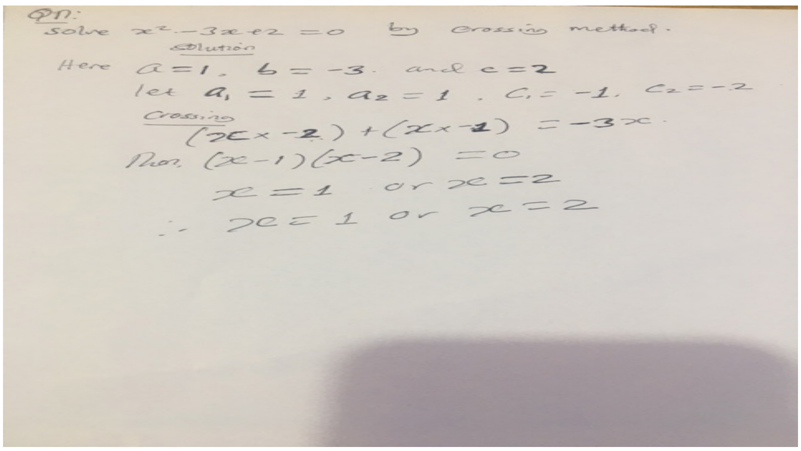


Figure 5. Quadratic equation created and solved by Student 3 using the crossing method.

From Figure 1, Student 1 created and solved the quadratic equation $4x^2 - 20x + 25 = 0$ and obtained two values which are similar $x = \frac{5}{2}$ or $x = \frac{5}{2}$. From Figure 2, Teacher 1 created and solved the quadratic equation $2x^2 + 8x - 24 = 0$ and obtained two values which are $x = 2$ or $x = -6$. From Figure 3, Student 2 created and solved the quadratic equation $x^2 - 7x + 12 = 0$ and obtained two values which are $x = 4$ or $x = 3$. From Figure 4, Teacher 2 created and solved the quadratic equation $2x^2 + 9x + 10 = 0$ and obtained two values which are $x = -\frac{5}{2}$ or $x = -2$. From Figure 5, student 3 created and solved the quadratic equation $x^2 - 3x + 2 = 0$ and obtained two values which are $x = 1$ or $x = 2$. All students and teachers obtained the correct answers. These answers were similar to those obtained using other methods for solving quadratic equations, including the general formula, splitting the middle term, factorization, and completing the square.

The crossing method involves a series of teaching and learning activities that help both teachers and students understand and apply this approach to solving quadratic equations effectively. These activities focus on conceptual understanding, practical application, and problem-solving strategies.

Teachers introduce the concept of the crossing method by explaining how it differs from traditional methods such as factorization, completing the square, the quadratic formula, and graphing. They may use visual aids, diagrams, or digital tools to illustrate the method and help students grasp its significance. Additionally, teachers demonstrate step-by-step solutions by solving quadratic equations using the crossing method on the board or with digital tools, comparing results with traditional methods to reinforce accuracy and consistency.

To ensure students understand the method, teachers guide them through examples by assigning simple quadratic equations and providing support in identifying intersections and verifying solutions. They also facilitate group discussions where students work together to solve problems, discuss their approaches, and share their observations, enhancing peer learning and engagement. To assess comprehension, teachers provide exercises or quizzes and offer constructive feedback to clarify misconceptions and strengthen students' grasp of the concept.

On the students' side, they actively engage by observing and understanding the concept as explained by their teachers. They take notes, ask questions, and seek clarification to enhance their comprehension. Students practice solving quadratic equations using guided examples, applying the crossing method, and comparing their solutions with other methods. They collaborate in group work and peer discussions to reinforce understanding further and share different perspectives on problem-solving.

As part of their learning activities, students explore graphical representations using manual or digital graphing tools to visualize the application of the crossing method. They identify points of intersection and analyze their significance in solving quadratic equations. Additionally, students apply the method to real-world problems, working on word-based applications that require solving quadratic equations using the crossing method. Finally, they engage in self-assessment and reflection by completing independent practice exercises, identifying areas of difficulty, and seeking further clarification from teachers or peers.

Through these activities, both teachers and students can effectively adopt and apply the crossing method in solving quadratic equations, promoting conceptual understanding, engagement, and problem-solving skills.

5. Insights from Teachers' and Students' Perceptions About the Crossing Method

We gathered information using a questionnaire from teachers and students about the crossing method as an alternative approach for teaching, learning, and solving quadratic equations in Tanzanian secondary schools. This study aimed to assess students' and teachers' familiarity with this method, their perceptions of its effectiveness, and its potential application in secondary school mathematics. The subsections below present the findings based on teachers' and students' opinions.

5.1 Students' familiarity with methods for solving quadratic equations

All participants were familiar with traditional methods such as the general formula, factoring, the quadratic formula, and completing the square. However, the crossing method was relatively unknown before the session, indicating a gap in awareness and application, as shown in Figure 6.

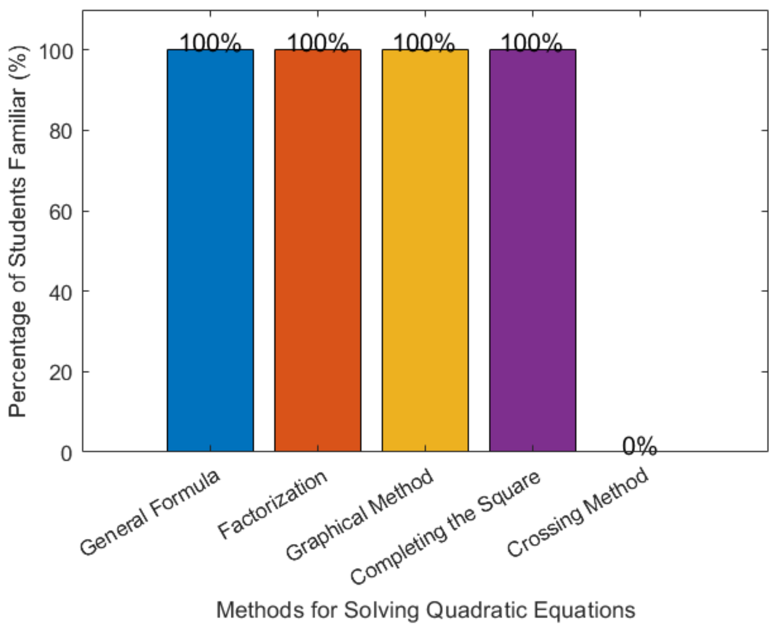


Figure 6. Students' familiarity with the crossing method.

5.2 Teachers' and students' prior knowledge of the crossing method

The majority of students (95%) and teachers (90%) had never encountered the crossing method before, suggesting that it has not been widely introduced in Tanzanian secondary schools. Figure 7 presents the results, highlighting the level of prior knowledge among students and teachers regarding the crossing method for solving quadratic equations. This underscores the need for further dissemination and integration of the method into teaching practices. The absence of the crossing method in mainstream teaching suggests that students may be missing out on an alternative problem-solving technique that could simplify quadratic equation solutions, particularly for those who struggle with conventional factorization. Integrating this method into the curriculum could provide learners with additional tools to approach quadratic equations, fostering a deeper conceptual understanding and improving problem-solving skills. Additionally, teacher training and the development of instructional resources are necessary to ensure effective implementation.

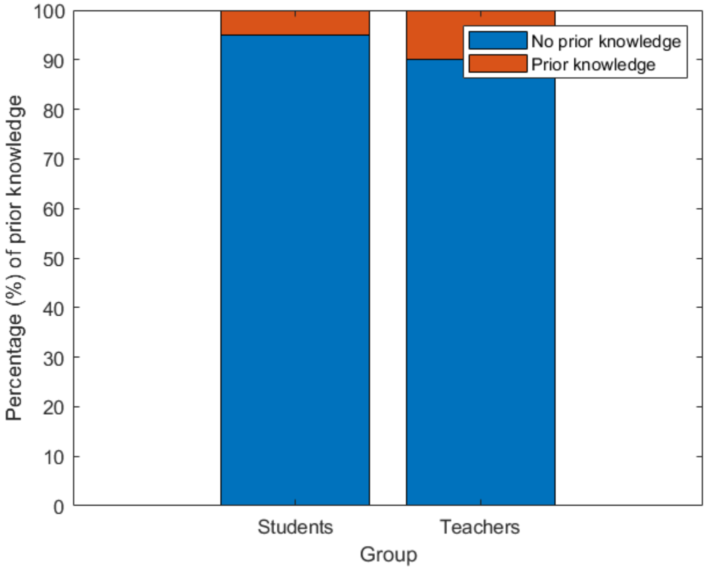


Figure 7. Students' and teachers' prior knowledge about the crossing method.

The effectiveness of any teaching approach depends significantly on the prior knowledge that both teachers and students have about the method. In the case of the crossing method, it is relatively new and not widely documented in existing mathematics curricula or textbooks. As a result, most teachers and students may have little to no prior knowledge of it before being introduced to the concept.

Many in-service and pre-service mathematics teachers are traditionally trained in solving quadratic equations using factorization, the quadratic formula, completing the square, and graphing. Since the crossing method has not been widely studied or included in standard instructional materials, most teachers may be unfamiliar with its concepts, applications, and effectiveness. Teachers with strong backgrounds in algebra and graphing techniques may find it easier to adopt and integrate the crossing method into their teaching practices. However, without formal training or exposure, they may be reluctant to rely on the method as an alternative approach.

Similarly, most students are introduced to quadratic equations through conventional algebraic methods stated earlier. Since the crossing method is not commonly taught, students are unlikely to have prior knowledge of this approach before it is introduced in the classroom. However, students who have experience with graphing equations and understanding points of intersection may adapt more quickly to the crossing method, as it involves recognizing graphical relationships between functions.

Given the lack of prior knowledge among both teachers and students, introducing the crossing method in secondary school mathematics requires proper instructional strategies, including teacher training, curriculum adjustments, and well-structured examples. Teachers need to be equipped with the necessary skills to explain and demonstrate the method effectively, and students need structured guidance and practice to understand its application.

Ultimately, while prior knowledge of the crossing method may be limited, its potential benefits as a simple, intuitive, and effective approach to solving quadratic equations highlight the importance of integrating it into the teaching and learning process.

5.3 Uniqueness of the crossing method

Respondents identified the crossing method as unique because it simplifies factorization without trial and error, making it more intuitive for some learners. Compared to traditional approaches, it offers a structured and visual way to identify the factors of a quadratic equation. Figure 8 presents respondents' views and percentages on the uniqueness of the crossing method.

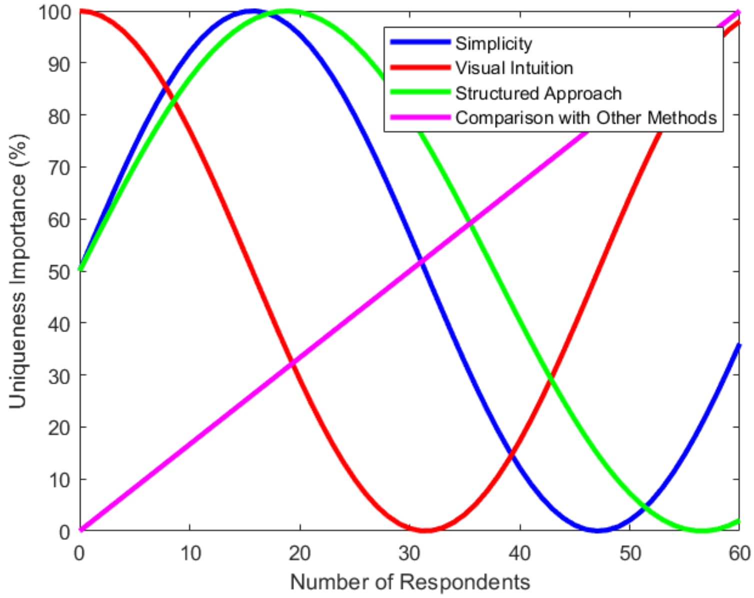


Figure 8. Respondents' opinions on the uniqueness of the crossing method.

As seen, the crossing method appears to produce more convincing results when compared to other methods. It shows more positive outcomes, further supporting its uniqueness.

The crossing method offers a unique approach to solving quadratic equations by incorporating both algebraic and graphical reasoning, making it distinct from traditional methods such as factorization, completing the square, the quadratic formula, and graphing. One of its key unique features is its ability to provide a visual and conceptual understanding of quadratic equations without solely relying on algebraic manipulation. Unlike the quadratic formula, which requires memorization, or completing the square, which involves complex steps, the crossing method simplifies the solving process and enhances comprehension.

Another unique aspect of the crossing method is its efficiency and time-saving nature in finding solutions to quadratic equations. It enables students to identify solutions by recognizing points of intersection, reducing computational effort while maintaining accuracy. Additionally, it accommodates quadratic equations with both positive and negative leading coefficients, offering flexibility in application.

Moreover, the crossing method serves as an alternative pedagogical tool that engages students actively by encouraging them to explore mathematical relationships dynamically. This method fosters critical thinking and problem-solving skills, as students can analyze and compare different approaches to finding solutions.

Due to its intuitive and straightforward nature, the crossing method can be a valuable addition to mathematics education, especially in classrooms where students struggle with abstract algebraic concepts. Its uniqueness lies in how it bridges the gap between algebraic and graphical techniques, making quadratic equations easier to understand and apply in real-world problem-solving.

5.4 Teachers ' and students' perceptions of using the crossing method for problem solving

Seventy percent of the respondents perceived the crossing method as appropriate for solving quadratic problems, while 20% believed it was not suitable, and 10% were undecided. The crossing method, as a problem-solving technique, was positively received by many students and teachers due to its simplicity and clarity. Students appreciated its step-by-step approach, which allowed them to visualize and better understand the process of solving quadratic equations. Teachers also found it useful as a teaching tool, as it helped engage students and facilitated a deeper understanding of how to approach quadratic problems, as shown in Figure 9.

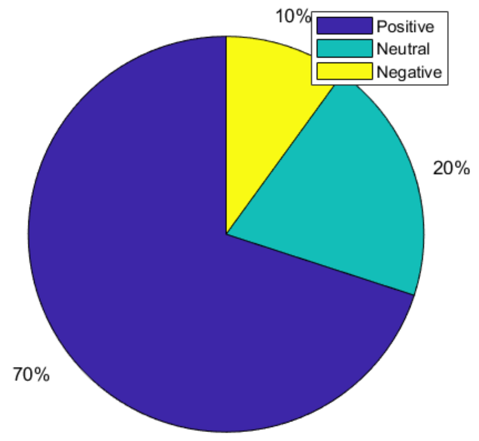


Figure 9. Teachers' and students' perceptions of using the crossing method for solving quadratic equations.

The perceptions of teachers and students toward using the crossing method for solving quadratic equations play a significant role in determining its effectiveness as a teaching and learning approach. Teachers' perceptions often revolve around the method's simplicity, efficiency, and ease of integration into the existing curriculum. Many teachers may appreciate that the crossing method provides a visual and conceptual understanding of quadratic equations, potentially reducing students' reliance on memorization and procedural approaches. Additionally, they may find it to be a useful alternative for students who struggle with traditional techniques such as factorization, completing the square, the quadratic formula, and graphing.

From the students' perspective, the crossing method may be perceived as a more intuitive and engaging approach to solving quadratic equations. Because it emphasizes visualization and logical reasoning, students may find it easier to grasp the concepts and apply them to different problems. The method's ability to produce accurate results quickly could also enhance students' confidence and motivation in learning mathematics. However, its effectiveness will depend on factors like students' familiarity with graphical representations and their exposure to alternative problem-solving strategies.

Overall, positive perceptions from both teachers and students could encourage the adoption of the crossing method as a complementary approach to existing techniques in mathematics education. Further research on its impact in classroom settings, including practical implementation and student outcomes, could provide deeper insights into how well the method supports conceptual understanding and problem-solving skills.

5.5 Applicability in teaching and learning

The crossing method is indeed an effective approach for teaching, learning, and solving quadratic equations. It enhances conceptual understanding and promotes engagement among both teachers and students.

By focusing on the intersection points of functions (such as linear and quadratic equations), the crossing method helps students visualize the solutions of quadratic equations graphically. This method reinforces algebraic techniques by allowing students to see the relationship between the equation and its graphical representation.

The increased engagement comes from interactive learning, where students actively explore quadratic equations by plotting graphs and interpreting intersections. Additionally, the crossing method encourages critical thinking as students must analyze and verify their solutions through multiple representations—graphical and algebraic.

If Figure 10 illustrates this concept, it likely depicts how students' comprehension improves by visualizing the intersections of curves corresponding to quadratic expressions. This visual approach clarifies abstract algebraic manipulations, making learning more intuitive and effective.

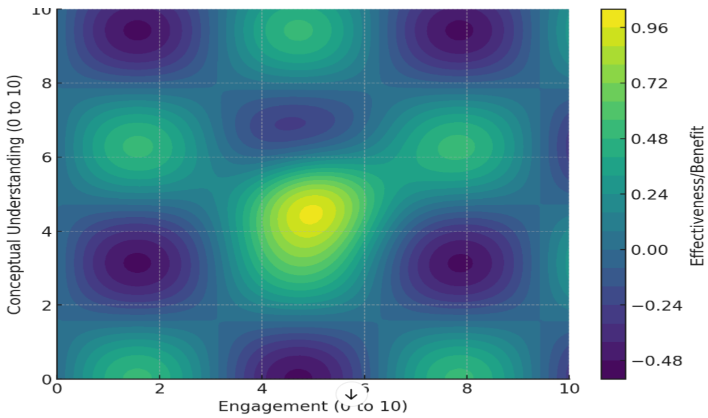


Figure 10. The crossing method is effective for conceptual understanding and engagement in teaching and learning quadratic equations.

6. Conclusion

This paper explores factorization, completing the square, the quadratic formula, graphing, and the crossing method as approaches that mathematics teachers can use to teach students how to solve quadratic equations. The findings indicate that all methods yield similar results, including the alternative crossing method. Based on this, the paper recommends incorporating the crossing method as an additional approach for teaching quadratic equations in ordinary-level secondary schools in Tanzania and beyond. This addition is crucial for expanding the existing knowledge base on teaching and solving quadratic equations alongside the four traditional methods currently included in Tanzanian mathematics textbooks and syllabi. Furthermore, we suggest that this study be extended to other mathematical concepts, such as solving systems of linear and polynomial equations, to explore the broader applicability of the crossing method.

While the crossing method offers a simple and intuitive approach to solving quadratic equations, it has certain limitations that may affect its effectiveness in some learning environments. These limitations should be considered when integrating the method into teaching and learning.

One key limitation is that the crossing method may not always be the most efficient approach for solving complex quadratic equations. For equations with irrational or complex roots, identifying points of intersection graphically or algebraically can be challenging, making the quadratic formula a more suitable option. Additionally, this method works well for equations that can be easily visualized or manipulated algebraically, but it may not be as effective for equations with cumbersome coefficients or those that require extensive simplification.

Another drawback is that students with weak graphing skills may struggle to understand or apply the method effectively. Since the crossing method often involves recognizing intersections between expressions, students who are unfamiliar with graphing techniques may find it difficult to interpret the visual or numerical relationships correctly. This can make it less accessible for learners who are more comfortable with algebraic manipulation rather than graphical reasoning.

Furthermore, the crossing method is not widely recognized or included in formal curriculum materials, meaning that teachers and students may have limited resources for learning and practicing the approach. Unlike established methods such as factorization, completing the square, and the quadratic formula, which are extensively covered in textbooks and examinations, the crossing method may not be explicitly taught in many educational systems, leading to hesitation in its adoption.

Additionally, while the method may be effective for quadratic equations with real and distinct roots, it may not be the most suitable for quadratic equations with complex conjugate roots, which do not have real intersection points. In such cases, algebraic methods like completing the square or using the quadratic formula are necessary to obtain accurate solutions.

Lastly, the generalizability of the crossing method to all types of quadratic problems remains limited. While it provides an alternative approach that can enhance conceptual understanding, relying solely on this method may not address all students' needs, particularly those who prefer analytical methods over graphical or numerical interpretations. While the crossing method presents an alternative and engaging way of solving quadratic equations, its potential drawbacks—such as difficulties in handling complex or irrational solutions, the need for strong graphing skills, limited formal recognition, and challenges with non-real solutions—should be considered. Teachers should incorporate the crossing method alongside traditional solving techniques to ensure a well-rounded approach to quadratic equations in mathematics education.

Despite the limitations presented above, the crossing method still plays a significant role in enhancing the teaching and learning of quadratic equations by providing an alternative approach that simplifies problem-solving. One of its key advantages is that it offers a conceptual and visual understanding of quadratic equations, helping students grasp the relationship between algebraic expressions and their graphical representations. This approach can make learning more engaging and intuitive, especially for students who struggle with abstract algebraic manipulations using traditional methods like factorization, completing the square, and the quadratic formula.

Another important aspect of the crossing method is that it is simple and efficient, allowing students to find solutions quickly compared to some conventional approaches. By focusing on points of intersection, students can determine roots without necessarily relying on lengthy calculations, making the method time-efficient for solving quadratic equations. This feature is particularly beneficial for students who find algebraic manipulation challenging and need an alternative way to verify their answers.

The crossing method also promotes active learning and engagement in the classroom. Since it encourages students to visualize equations and explore mathematical relationships, it fosters critical thinking and problem-solving skills. Additionally, teachers can use it as an interactive teaching tool to help students compare different methods of solving quadratic equations, reinforcing a deeper understanding of key mathematical concepts.

Moreover, this method can serve as an alternative pedagogical tool for mathematics instruction. By integrating the crossing method alongside traditional approaches, teachers can provide students with multiple strategies for solving quadratic equations, catering to different learning styles. Some students may find graphical methods easier to understand, while others may prefer algebraic techniques, making this alternative method a valuable addition to mathematics education.

Additionally, the crossing method has the potential to expand problem-solving strategies beyond quadratic equations. It can be applied to solving systems of equations and may help students develop a foundation for more advanced mathematical concepts. Its flexibility in working with both positive and negative leading coefficients also makes it a versatile tool for different quadratic problems. The importance of the crossing method lies in its ability to enhance conceptual understanding, improve problem-solving efficiency, promote active learning, and serve as an alternative approach in mathematics education. By incorporating this method into the curriculum, students and teachers can gain a richer understanding of quadratic equations while fostering deeper mathematical reasoning and engagement.

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