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AHP – TOPSIS Model as a Mathematical Support in the Selection of Project from Aspect of Mobility – Case Study

Aybeyan Selimi[1], Mimica Milošević[2], Muzafer Saračević[3]

[1]PhD Candidate, International Vision University, Faculty of Informatics, Major C. Filiposki Nr.1, Gostivar, Macedonia
[2]Faculty of Business Economics and Entrepreneurship, Mitropolita Petra 8, 1100 Belgrade, Serbia
[3]Associate Professor, International University of Novi Pazar, Department of Computer Science, Dimitrija Tucovica bb. 36 300 Novi Pazar, Serbia

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***Corresponding author**: Aybeyan Selimi, PhD Candidate International Vision University, Faculty of Informatics, Major C. Filiposki Nr.1, Gostivar, Macedonia
Email: aybeyan@vizyon.edu.mk

Abstract

The multi-criteria decision support systems are used in various fields of human activities. In each alternative of the multi-criteria decision making, the problem could be represented as a set of properties or constraints. Today the modern societies demand a high degree of mobility in the process of planning and construction, therefore it is necessary to examine a large number of different criterions in the decisions. The mobility is an important feature of life and plays a significant role in people's activities. This paper presents a model for the selection of projects of the housing complexes, which can be integrated into the urban core of the cities and in such case that encourages frequency of activities and events of employed in IT sector, while increases the quality of life in the city. The Comparison of predetermined criteria and determination of their weight has been done with AHP method, and the best alternative is selected with TOPSIS method. The aim of this study is to highlight the importance of using of the multi-criteria optimization in the moments of making complex and important decisions by taking into account the number of criteria that need to be considered in the course of construction practice.

Keywords

AHP method, comparison matrix, TOPSIS method, eigenvector.

1. Introduction

Civil engineering is one of the largest industries in any country. It provides a significant contribution to the economy of one country and employs a huge number of people. That is why projects in this industry have made a major change in the management strategy and in the implementation time. Particular importance is given to mobility which is very important and must involve in projects today. Due to this each company has a strategic plan for mobility and develops a plan to effectively bridge the barrier of mobility. Mobilizing barriers represent the difficulty of moving for companies from one strategic group to another. For this reason, companies can not compete with competition with a similar market profile, but also with those who sell products at the same price and give the same quality. For companies and people, mobility today it becomes a very important need both in business and in private life. The IT sector requires new computer structures that are mobile, malleable and available at any location. Therefore new types of networks and architecture of electronic technologies that have mobile computing capabilities are being developed. From the perspective of software engineering, mobile computing is the study of systems in which computer components can change the location. Mobility improves productivity and profits, and therefore has a wide use in all business sectors. The benefits of mobility are: better communication, improved company efficiency, improved workflow, etc. With better mobility, businesses and people save money and time and have more efficiency in managing their business. Mobility has completely transformed the sense of decision making in the construction sector. It helps to reduce job downtime and provides better insight into jobs and helps in making decisions. When there is more than one alternative, the choice of the best alternative is done as a problem of multi-criteria decision making. Methods of more criteria decision making make it easier to make a decision in situations where there are many different alternatives, which can often be contradictory to each other. In many parts of the construction sector, questions are raised: How to make the selection of the best offer? The selection of the best offer consist the various factors and circumstances of the construction. The mobility of a residential object has a direct impact on the satisfaction of the household, and therefore plays an important role in the selection of construction projects. The complexity of project selection has different aspects, and in the selection are installed the different criteria. Selection of the best alternative, decision-makers can do only with complex scientific and analytical procedures. The complexity of the problem imposes the need for creating mathematical models where methods will help decision makers in analyzing and selecting solutions [17]. For this purpose, in this paper, a mathematical model for the selection of the best project is constructed using methods of multi-criteria decision making. The decision maker implicitly reserves the right to accept, modify or reject a solution obtained on the basis of a mathematical optimization model [8]. AHP method as a multi-criteria method has wide application in the decision in the construction sector [15]. In this paper, we have identified several effective criteria that influence the process of selection of the best offer.

1.1 Aim of paper

The main goal is to select the project with the best mobility in construction based on the assessment and the proposed comparison of the alternatives. Until the selection, the decision-maker can only come by using complex scientific analytical procedures. To that end, in this paper is constructing the AHP-TOPSIS method.

2. Research Methodology

Construction of the model for selection of project from the aspect of mobility was made with the combination of the AHP - TOPSIS method and consists of four steps.
1. Defining the problem and determining the goal
2. Determining the criteria used in the model.
3. Determination of the weight coefficients of the criterion
4. Evaluation of the alternative.

In the first step, the criteria that are applied in the selection of the project with the best mobility are selected. In the second step, the weight coefficients of each criterion with the AHP method were determined. The final selection was done in the third step with the TOPSIS method. Construction of the model has been made taking into account the following priorities:

- The project in the first place must have the safety, health, and welfare of human life
- The project must take account the sustainable development
- They have to avoid conflicts of interest between the seller and client
- Planning, design, and construction should fulfill the aim of the potential client
- Cooperation in project design with the other field expert
- Completed project must have optimal cost
- Construction of the project is planned in the interests of energy efficiency.

3. Case Study

In the construction of each building facility, some basic requirements must be met. The basic requirements imposed on the building are safety, mechanical resistance and stability, protection of human health, protection in case of fire, the mobility of the building facility, protection against corrosion and the external influences. Safety is reflected in the ability of buildings to withstand all the predicted influence that occur during normal use. The building should be designed so that during construction, damage to other buildings will not be caused and the safety of construction workers will be taken into account [21]. The issue of mobility has an important role for families and is viewed as a satisfaction that provides housing.

Criteria for selecting a project with the best mobility have been established. In this case, five projects A_1, A_2, A_3, A_4, and A_5 are assessed on the basis of 9 criteria. When selecting the criteria, the following characteristics were taken into account:

- The importance of mobility as a basic indication of satisfaction in the home for the household.
- Functionality (premises, access, organization, content, context and general functionality of an architectural unit of an object is a key factor that technically perfects one object).
- Aesthetic design and contemporary architectural expression of objects (apart from what needs to be answered by the needs of a modern man, should contribute positively to the socio-psychological development of people living in such environments).
- Energy efficiency of the facility (generally refers to devices, measures, and behavior of energy efficiency).
- The location of the object (affects mainly the value of the object and the quality of life in the object).
- The constructive system (the corresponding construction system allows for a long lifetime of the structure of the building, provides favorable conditions for possible adaptations in appearance and the sense of security).
- Environment (has an impact on the quality of life).

The criteria for selecting a construction project from the aspect of mobility have been established and the following tags were introduced:

Table 1. Tags used in the model [source - authors according to research results]

A	Appearance
C	Construction cost
D	Durability
O	Power outlets
M	Mobility
L	Low Maintenance
P	Privacy
S	Sound transmission
V	Visibility

3.1 Analytical Hierarchy Process (AHP)

Decision making is a process that is continuously taking place in all segments of life and by every person. Linear programming represents the basis of multi-criteria decision making, which first began to develop as a target programming [5]. Because of the nature of the problem of multi-criteria decision making, it is important to determine the goal that is essential for the decision. The main characteristic of multi-criteria methods (MCDM) is decision making based on decision-making problem. Deciding involves more than one criteria/ attributes/factors for subordination and selection of the best alternative. Criteria/attributes/factors are instruments for assessing choices. Their main quality lies in the fact that they can find a compromise solution between groups of different interests, giving ranked options as the final result. Priorities are given to certain criteria, depending on the interests and needs of different groups, influence the ultimate decision in the system. Decision support systems are interactive computing systems that aim to help decision-makers to identify, structure and/or solve semi-structured and unstructured problems and to make a selection among alternatives. The process of the analytical hierarchical method (AHP) was constructed by Thomas L. Saaty in the early 1970s, based on a set of criteria and attributes for each alternative.

3.1.1 Methodological basis of the AHP method

The analytical hierarchical process (AHP) falls into the class of methods of the "soft" optimization and consists of the following levels:

- Interactive Creation and Hierarchical Presentation of Criteria (Goals)
- Pair-wise comparison of criteria and alternatives in the $n \times n$ comparison matrix using the scale shown in Table 2.
- In the $n \times n$ comparison matrix, there is a total of $n(n-1)/2$ comparisons. Each element in pairs is compared to each element directly at the higher level.
- Pair-wise evaluation of elements in the hierarchy (goals, criteria (sub-criteria) and alternative) goes top-down direction.

- Consistency is determined by the maximal eigenvectors λ_{max} with the calculation of the consistency index from equality $CI = (\lambda_{max} - n)/(n - 1)$. The consistency of the decision is obtained with consistency ratio CR. If in comparison matrix is CR <0.10, then the estimates of the relative importance of the criteria (priorities of the alternative) are counted as acceptable. Otherwise, one must find the reasons why the inconsistency of the assessment is unacceptably high.
- Synthesis of all evaluations and subordination of the alternatives according to a strictly determined mathematical model.

3.1.2 Interactive Creation and Hierarchical Presentation of Criteria (Goals):

The most important feature of the AHP method is that the decision makers consider deciding issues as elements that have a mutual hierarchical relationship. The highest level of this hierarchy is the main goal of the decision maker. The main objectives are subordinate to the criteria that are important in decision making [2]. In the lowest level of the hierarchy are the alternatives.

3.1.3 Pair-wise comparison of alternatives and criteria

In the second level, the criteria and alternatives are compared in pairs, the comparison is made directly with the superior element at a higher level [11]. This pairing is shown as a square matrix. Any comparison in the square matrix represents the hierarchy of each element of the first column based on the elements in the rows above them. The comparisons provide the answer to the following two questions. Which of the two elements are more important in relation to the criteria that are at a higher level? What is the level of importance? The character table is given in Table 2.

Table 2. Saaty's scale of importance [14]

Intensity of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favor one activity over another
5	Strong importance	Experience and judgment strongly favor one activity over another
7	Very strong or demonstrated importance	An activity is favored very strongly over another, its dominance demonstrated in practice
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2,4,6,8	Intermediate values	Requires compromise or further division
Reciprocals of above	If factor *i* has one of the above numbers assigned to it when compared to factor *j*, then j has the reciprocal value when compared with *i*	A reasonable assumption

Table 3. Values of a random index RI [14]

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0.00	0.00	0.58	1.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

3.2 Mathematical concept of TOPSIS method

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method is developed by Hwang and Yoon in 1981 as a method of multi-criteria analysis where alternatives are evaluated based on their distance in relation to the positive ideal solution and negative ideal solution [5]. The best alternative is in the shortest distance from the positive ideal solution and in the longest distance from the negative ideal solution. The application of the method is carried out in the following steps:

Step 1: For the application of the method, we first form a decision matrix D. Rows of decision matrix are alternatives, and the columns are the criteria.

$$D = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & & \dots & \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix}$$

Step 2. To normalize the decision matrix we use the method of normalization of the vector. With vector normalization, the direction of the criterion does not change. Normalization of the decision matrix is obtained with:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \; i = 1, 2, \dots, m, \; j = 1, 2, \dots, n$$

Step 3. We calculate the weighted normalized decision matrix. From the decision maker, the weight of each criterion must be determined. The weighted normalized matrix $V = (v_{ij})_{m \times n}$ is obtained as a product of the normalized matrix with the weighted matrix of the criterion

$$V = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \dots & w_n r_{1n} \\ w_1 r_{21} & w_2 r_{22} & \dots & w_n r_{2n} \\ \dots & & \dots & \\ w_1 r_{m1} & w_2 r_{m2} & \dots & w_n r_{mn} \end{bmatrix}$$

where $W = (w_1, w_2, \dots, w_n)$ is a set of weight so that $\sum_{i=1}^{n} w_i = 1$.

Step 4. We determine the positive ideal solution and negative ideal solution. If the criterion has a positive impact, then the positively ideal solution consists of the highest value in each criterion, while the negative ideal solution has the smallest value in each criterion. The positive ideal solution and the negative ideal solution are defined in the following way:

$$A^* = \left\{ \left(\max_i v_{ij} \mid j \in J_+ \right), \left(\min_i v_{ij} \mid j \in J_- \right), i = 1, 2, \dots, m \right\} = \left\{ v_1^*, v_2^*, \dots, v_n^* \right\}$$

$$A^- = \left\{ \left(\min_i v_{ij} \mid j \in J_+ \right), \left(\max_i v_{ij} \mid j \in J_- \right), i = 1, 2, \dots, m \right\} = \left\{ v_{1-}, v_{2-}, \dots, v_{n-} \right\}$$ where,

$J_+ = \{ j = 1, 2, \dots, n \}$ - *j* associated with the criteria having a positive impact

$J_- = \{ j = 1, 2, \dots, n \}$ - *j* associated with the criteria having a negative impact.

Step 5. In this step, we calculate the separation measure (Euclidean distance) to each alternative. The distance between the target alternative *i* and the negative ideal solution is:

$$S_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_{j-} \right)^2}, \; i = 1, 2, \dots, n$$

and the distance between the alternative i and the positive ideal solution

$$S_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^*\right)^2},\ i = 1, 2, \ldots, n$$

Step 5: Calculate the relative closeness to the positive ideal solution

$$C_i = \frac{S_i^-}{S_i^- + S_i^+},\ 1 \geq C_i \geq 0,\ i = 1, 2, \ldots, m$$

Step 6: Rank the preference order

$$C_i = 1 \Rightarrow A_i = A^*$$

$$C_i = 0 \Rightarrow A_i = A^-$$

4. Results

AHP - TOPSIS model for selecting a project from the aspect of mobility consists of the following steps:
Step 1: On the basis of the evaluation of the criteria, was formed a comparison matrix:

Table 4. Comparison matrix [source - authors according to research results]

	M	**D**	**L**	**O**	**S**	**A**	**P**	**V**	**C**	***w***
M	1	2	3	3	2	4	3	7	8	0.282639
D	1/2	1	2	3	2	2	3	7	4	0.180864
L	1/3	1/2	1	3	2	3	2	5	2	0.130491
O	1/3	1/3	1/3	1	2	3	2	3	2	0.0969843
S	1/2	1/2	1/2	1/2	1	3	2	3	5	0.108399
A	1/4	1/2	1/3	1/3	1/3	1	2	3	2	0.0654208
P	1/3	1/3	1/2	1/2	1/2	1/2	1	3	2	0.0662909
V	1/7	1/7	1/5	1/3	1/3	1/3	1/3	1	1/2	0.0296537
C	1/8	1/4	1/2	1/2	1/5	1/2	1/2	2	1	0.0392576

Step 2. For this comparison matrix using the AHP method, we obtain a vector of weight coefficients given in the last column on right in Table 4.
Step 3. Evaluation of the alternatives was done with the TOPSIS method and on this basis, in this step, the criteria of positive and negative impact were determined as in Figure 1.

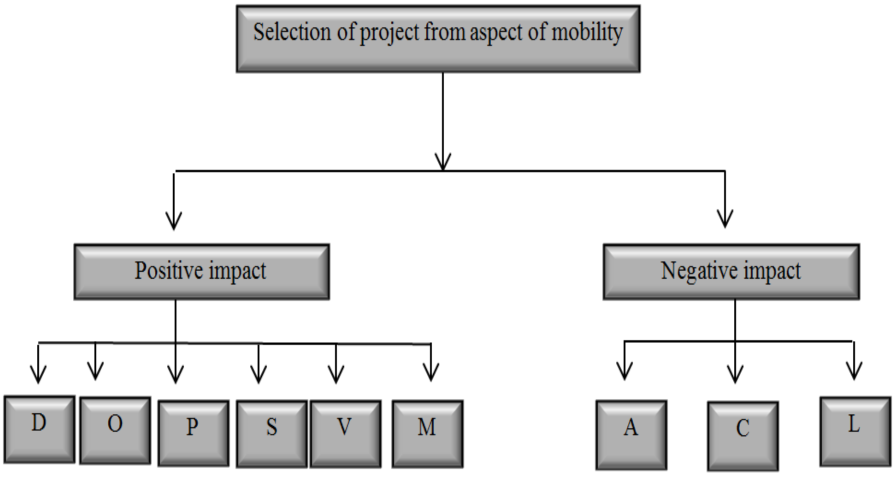

Figure 1. Selection of best alternative [source - authors]

Step 4. The goal of the decision maker is to maximize the positive and minimize the negative effects of the selection. To this end, a decision matrix was established with 9 criteria and 5 alternatives in the following way:

Table 5. Decision matrix [source - authors according to research results]

	M	**D**	**L**	**O**	**S**	**A**	**P**	**V**	**C**
A_1	9	7	7	5	5	1	7	3	5
A_2	9	7	5	5	1	1	5	3	3
A_3	9	7	7	5	5	5	3	1	3
A_4	9	7	5	1	3	1	5	3	5
A_5	9	7	5	3	5	3	3	5	3

Step 5. With the method of normalizing the vector we obtain a normalized form of the decision matrix:

Table 6. Normalized matrix [source - authors according to research results]

	A_1	**A_2**	**A_3**	**A_4**	**A_5**
M	0.447214	0.447214	0.447214	0.447214	0.447214
D	0.447214	0.447214	0.447214	0.447214	0.447214
L	0.5322	0.380143	0.5322	0.380143	0.380143
O	0.542326	0.542326	0.542326	0.108465	0.325396
S	0.542326	0.108465	0.542326	0.325396	0.542326
A	0.164399	0.164399	0.821995	0.164399	0.493197
P	0.64715	0.46225	0.27735	0.46225	0.27735
V	0.412082	0.412082	0.137361	0.412082	0.686803
C	0.569803	0.341882	0.341882	0.569803	0.341882

Step 6. In this step, the normalized decision matrix was weighted and is obtained the weighted matrix:

Table 7. Weighted matrix [source - authors according to research results]

	M	**D**	**L**	**O**	**S**	**A**	**P**	**V**	**C**
A1	0.1264	0.08088	0.06945	0.0526	0.05879	0.01075	0.0429	0.01222	0.02237
A2	0.1264	0.08088	0.04961	0.0526	0.05879	0.01075	0.03064	0.01222	0.01342
A3	0.1264	0.08088	0.06945	0.0526	0.05879	0.05377	0.01838	0.00407	0.01342
A4	0.1264	0.08088	0.04961	0.01052	0.03527	0.01075	0.03064	0.01222	0.02237
A5	0.1264	0.08088	0.04961	0.03316	0.05879	0.03226	0.01838	0.02036	0.01342

Step 7. Calculating the positive ideal and the negative ideal solution:

Table 8. Positive ideal and negative ideal solution [source - authors according to research results]

	M	**D**	**L**	**O**	**S**	**A**	**P**	**V**	**C**
A^*	0.12 64	0.0808 849	0.049 605	0.052 597	0.058 787	0.053 775	0.0429 002	0.0203 663	0.0134 215
A^-	0.12 64	0.0808 849	0.069 447	0.010 519	0.035 272	0.010 755	0.0183 858	0.0040 732	0.0223 691

Step 8. In this step, is calculated the distance to each alternative and are obtained the following results:

Table 9. Distance to every alternative [source - authors according to research results]

Alternative	S_i^+	S_i^-
A_1	0.0232408	0.0695814
A_2	0.0147175	0.0697468
A_3	0.0557754	0.049026
A_4	0.0511874	0.0496092
A_5	0.0388108	0.0468776

Step 9. For the relative closeness of the alternative to the ideal solution, are obtained the following results:

Table 10. Relative closeness of alternatives to the ideal solution [source - authors according to research results]

Alternative	C_i
$\mathbf{A_1}$	0.74962
$\mathbf{A_2}$	0.825755
$\mathbf{A_3}$	0.467799
$\mathbf{A_4}$	0.492171
$\mathbf{A_5}$	0.547071

5. Conclusion

Matrix operations are done in the software package Mathematica. The AHP - TOPSIS model is applied for selection of a project from the aspect of mobility. Precise implementation of the procedure in the TOPSIS method has resulted in the second alternative A_2 having the highest total value of 0.825755, which is why it is the most favorable alternative for selecting a project with the best mobility. The proposed AHP - TOPSIS model can be successfully applied in decision making in various segments of construction. Mathematical and thus objective methods that support the decision-making process in construction, undoubtedly help in everyday construction work. Proposed AHP - TOPSIS model can be successfully applied in decision-making in different segments of life and can serve as a basis for the development of models for selection of the living space for people with disabilities.

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