



A Given Cube Edge Can Be Used to Make Twice Cube Edge

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Abstract

After more than two thousand years of research and demonstration by our forefathers, it has been established that a cubic equation with rational coefficients has no rational root, and then a line segment with a length equal to any real root cannot be constructed using only a ruler. Based on this, it is concluded that "cubic product belongs to ruler drawing and cannot be a problem." This drawing does not involve a cubic equation, but through exploration, it is found that the edge ratio of a given cube to a double cube is fixed. If the ratio of two edges can be found, it is possible to make a double cube of a given cube with a given edge. If we need to find the ratio of the edge length of a known cube to the edge length of a double cube, we need to use the edge length of the known cube to determine the edge length of the double cube, and then construct the double cube based on the dimensions of the known cube.

Keywords

Ruler drawing; Cubes; Edge; Two-edge ratio

1. Introduction

The reason why the introduction "Cubic product belongs to the problem that ruler can't draw" is that the edge line segment of a known cube is the only known condition. The cubic equation with a twice-known cubic edge line segment has no rational root, so the line segment with a twice-known cubic edge can't be drawn with a ruler. This is the theoretical basis that the ruler can't make a line segment equal to the edge of the cube. On the premise of admitting that a ruler can't make the theory of a double cube, this paper still insists on exploring the drawing of a double cube with a ruler. It's not that we don't believe in the theory that a ruler can't draw, but that we explore the drawing method of the edge line segment of a double cube with a ruler by drawing method. Although the edge of a known cube is an irrational root that cannot be made with a ruler, it is not difficult for the ruler to make a line segment equal to the multiple ratios of the known edge. If we know the ratio of the known cube edge to twice the cube edge, we can use the ruler to make a line segment equal to twice the cube edge according to the ratio of the known cube edge to twice the cube edge. Therefore, our research is not to directly make the irrational root line segment of the cube with a ruler but to explore how to find the ratio of a given cube edge to twice the cube edge. Although there is no ready-made method to find the ratio of the two at once, it is possible to try to find the ratio of the two one by one with the original method. For example, if the known cube is 1, the edge of the known cube is also 1, and the ratio of the edge of the 2-fold cube to the known edge must be several times 1:00, so the attempt of the ratio of the two directly starts from the second digit. First, set the ratio of the two to 1:1.1, and then set the ratio to 1:1.2 after calculating that the cube of 1.1 is obviously less than 2; After calculating that the cube of 1.2 is 1.728, the ratio is 1:1.3, and the cube of 1.3 is 2.197, it can be confirmed that the ratio of two sides is 1:1.2. After many attempts, the ratio of the two sides is 1:1.25, and the cube of 1.25 is calculated as; It is still less than 2 times cubic 2, and the cube of 1.26 is 2.000376 after the ratio is increased to 1:1.26. Although the calculation result of the ratio of edges to edges of 1:1.26 is larger than that of the double cube, the difference between them is only a little over ten thousandths [1-3]. Based on this, it

is considered that the ruler can make a cube with a volume very close to that of a given volume. In order to prove that the ratio of two edges is suitable for the edges of any given cube, cube numbers with different sizes are calculated and tested. Compared with the double cube, the difference between the cubes with edges calculated by this ratio is a little over ten thousandths. For example, let a given cube be 1.2 cubic meters, a twice cubic meter be 2.4 cubic meters, the edge of the 1.2 cubic meter be 1.062658569, and the edge of the twice cubic meter be 1.062658569×1.26 , and the difference with the given cubic meter is also a few ten thousandths; Let a given cube be a cube, a cubic cube be a cube, and the edges of the cube are, and the edges of the cubic cube are different from the times of the given cube by a few ten thousandths. Drawing: See Try Drawing 1a.

$$2221.953125 \sqrt[3]{1.338949797} = 2.400451222.410.5210.50.7937005262 = 0.793700526 \times 1.26 = 1.000062663 \sqrt[3]{1.000062663} = 1.0001880.5211.$$

It is known that the edges of a given cube are equal to each other in the world and a line segment with a radius of a point as the center of the circle is used to extend the arc to the point. The volume of a cube (see Figure 1b) made with the line segment as the edge is several ten-thousandths of the volume of the given cube. Based on this, it is proved that it is possible to continue to try to compare the two edges more accurately and make the difference between the two cubes of a given cube smaller until they are equal by this drawing method. Then continue to try to increase the number of digits of the ratio of two edges for comparison, and through a large number of figures of the ratio of two edges, try to find that the ratio of two edges can be obtained without trying, that is, the edge of a given cube is equal to the edge of a given cube.

$$\overline{ZJ} \overline{JG} \overline{ZJ} 0.26 \overline{JG} \overline{ZJ} \overline{CZ} \overline{ZC} 1221: 1.25992105$$

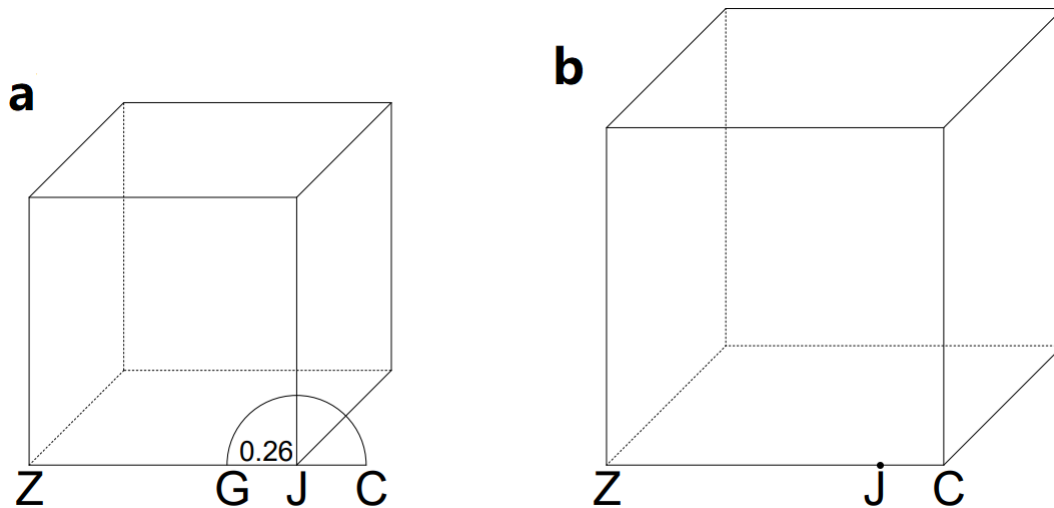


Figure 1. A volume doubling attempt diagram (a) of a known cube; (b) try to figure out.

Then, through the digital calculation of several cubes, it is proved that the cube with the line segment equal to the edge of the given cube made by the ruler is twice as large as the given cube. Give examples to prove, second, the theory proves that:

2. The first case

It is known that cubic volume, edge, $= \sqrt[3]{0.5} = \sqrt[3]{0.5} = 0.793700526$.

It is known that the cube edge is twice the cube edge $= 1: 1.25992105$.

Seek: a cube with twice the volume $= \sqrt[3]{0.52}$.

Drawing: Make a volume, as shown in Figure 2a. $= \sqrt[3]{0.5}$.

- 1) Which is equal to the edge of a given cube (Figure 2b), \overline{ZJ}
- 2) Do it on the line segment, so that $\overline{ZJ} \overline{JG} \overline{ZJ} = 0.25992105$
- 3) Take the point as the center and the radius as the arc intersection extension line to the point, $\overline{JG} \overline{ZJ} \overline{C}$
- 4) Use the cube made for the edge as the required cube (Figure 2b). \overline{ZC}

Proof: Figure 2b is twice the size of the cube in Figure 2a.

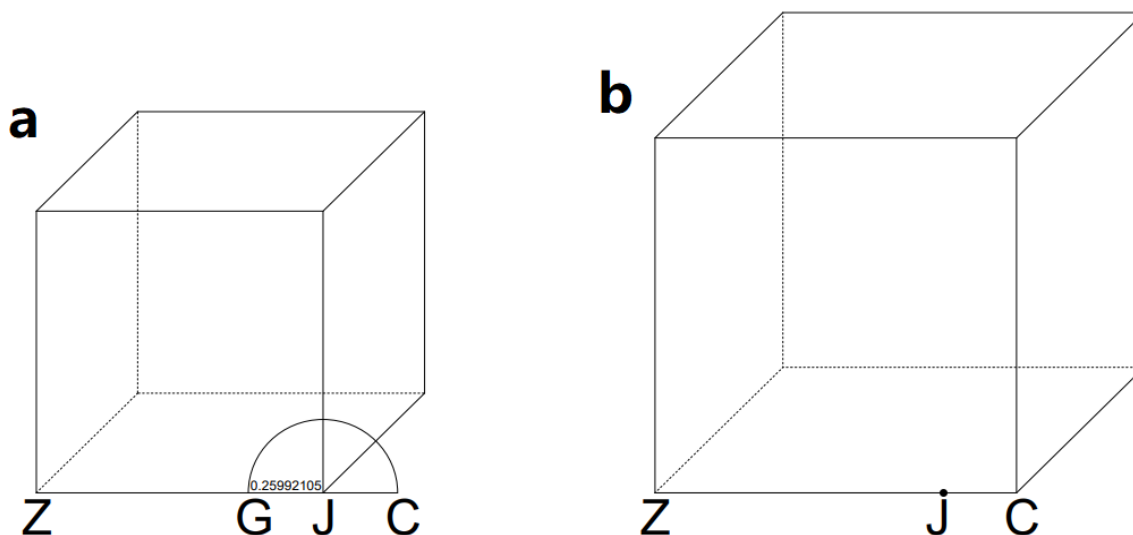


Figure 2(a). Volume doubling proves that the known cube.

Figure 2(b). Certification chart.

Prove:

∴ The edge of the cube is known, and the known edge is twice the edge of the cube. $\overline{ZJ} = 0.793700526ZJ: \overline{ZC} = 1: 1.25992105$

∴ $\overline{ZC} = 0.793700526 \times 1.25992105 = 1$

∴ \overline{ZC} is the edge of the cube, $\overline{ZC} = 1$

∴ \overline{ZC} Cube = $1^3 = 1$

∴ \overline{ZC} A cube with an edge is equal to, it is known that the cubic meter of a cube is equal to a cube, 10.521

∴ A cube with edges is equal to the times of a given cube.

3. The second case

It is known that cubic volume, edge, = $\sqrt[3]{1.6} = \sqrt[3]{1.6} = 1.169607095$

Known edge times cube edge: $2 = 1: 1.25992105$

Find: a cube with twice the volume = $\sqrt[3]{1.62}$

Drawing: Make a cuboid, as shown in Figure 2a; = $\sqrt[3]{1.6}$

1) Which is equal to the edge of a given cube, as shown in Figure 2b; \overline{ZJ}

2) On the line segment, make; $\overline{ZJ} \overline{JG} \overline{JG} = 0.25992105$

3) Take the point as the center and the radius as the arc intersection extension line to the point; $\overline{JG} \overline{ZJ} \overline{C}$

4) Use the cube made for the edge as the required cube. \overline{ZC}

Proof: Figure 2b is twice the size of the cube in Figure 2a.

Prove:

∴ The known edge of the cube is 1.169607095, and the known edge is times the cube edge. $\overline{ZJ} = \overline{ZJ}: 2\overline{ZC} = 1: 1.25992105$

∴ $\overline{ZC} = 1.169607095 \times 1.25992105 = 1.473612599$

∴ \overline{ZC} is the edge of the cube, $\overline{ZC} = 1.473612599$

∴ \overline{ZC} Cube = $1.4736126^3 = 3.1999999998$

∴ \overline{ZC} A cube with edges is equal to 99999998, and it is known that the cubic meter of a cube is equal to a cube.

∴ \overline{ZC} A cube with edges differs from a given cube by several tens of billions.

4. The third case

Known: cubic meter volume = $\sqrt[3]{3}$ Edge, = $\sqrt[3]{3} = 1.44224957$

Known edge times cube edge: $2 = 1: 1.25992105$

Seek: cuboid of volume, = $\sqrt[3]{3}$

Drawing: Make a cuboid, as shown in Figure 2a; = $\sqrt[3]{3}$

- 1) Which is equal to the edge of a given cube, as shown in Figure 2b; \overline{ZJ}
 - 2) On the line segment, make; $\overline{ZJ}\overline{JG}\overline{JG} = 0.25992105$
 - 3) Take the point as the center and the radius as the arc intersection extension line to the point; $\overline{JJG}\overline{ZJ}C$
 - 4) Use the cube made for the edge as the required cube. \overline{ZC}
- Proof: Figure 2b is twice as large as the cube in Figure 2a.

Prove:

∴ The edge of a cube is known, and the known edge is times the edge of the cube. $\overline{ZJ} = \sqrt[3]{3} = 1.44224957\overline{ZJ}: 2\overline{ZC} = 1: 1.25992105$

∴ $\overline{ZC} = 1.44224957 \times 1.25992105 = 1.817120593$

∴ \overline{ZC} is the edge of the required cube, $\overline{ZC} = 1.817120593$

∴ \overline{ZC} Cube = $1.817120593^3 = 6.000000002$

∴ \overline{ZC} A cube with an edge is equal to, it is known that the times of the cube are equal to the cube, 6.000000002326

∴ \overline{ZC} A cube with edges differs from a given cube by several tens of billions.

5. The fourth case

Known: cubic meter volume, edge, = $\sqrt[3]{4} = \sqrt[3]{4} = 1.587401052$

Known edge times cube edge: $2 = 1: 1.25992105$

Seek: a cube with twice the volume, = $\sqrt[3]{42}$

Drawing: Make a cuboid, as shown in Figure 2a; = $\sqrt[3]{4}$

- 1) Which is equal to the edge of a given cube (Figure 2b); \overline{ZJ}
 - 2) On the line segment, make; $\overline{ZJ}\overline{JG}\overline{JG} = 0.25992105$
 - 3) Take the point as the center and the radius as the arc intersection extension line to the point; $\overline{JJG}\overline{ZJ}C$
 - 4) Use the cube made for the edge as the required cube. \overline{ZC}
- Proof: Figure 2b is twice as large as the cube in Figure 2a.

Prove:

∴ The edge of a cube is known, and the known edge is times the edge of the cube. $\overline{ZJ} = \sqrt[3]{4} = 1.587401052\overline{ZJ}: 2\overline{ZC} = 1: 1.25992105$

∴ $\overline{ZC} = 1.587401052 \times 1.25992105 = 2$

∴ \overline{ZC} is the edge of the required cube, $\overline{ZC} = 2$

∴ \overline{ZC} Cubic, = $2^3 = 8$

∴ \overline{ZC} A cube with an edge is equal to, it is known that the times of the cube are equal to the cube, 8428

∴ \overline{ZC} A cube with edges is equal to the times of a given cube.

6. The fifth case

Known: cubic meter volume, edge, = $\sqrt[3]{13.5} = \sqrt[3]{13.5} = 2.381101578$

Known edge times cube edge: $2 = 1: 1.25992105$

Seek: a cube with twice the volume, = $\sqrt[3]{13.52}$

Drawing: Make a cuboid, as shown in Figure 2a; = $\sqrt[3]{27}$

- 1) Which is equal to the edge of a given cube (Figure 2b); \overline{ZJ}
- 2) On the line segment, make; $\overline{ZJ}\overline{JG}\overline{JG} = 0.25992105$

3) Take the point as the center and the radius as the arc intersection extension line to the point; $\overrightarrow{JJGZ}C$

4) Use the cube made for the edge as the required cube. \overrightarrow{ZC}
 Proof: Figure 2b is twice the size of the cube in Figure 2a.

Prove:

\because The edge of a cube is known, and the known edge is times the edge of the cube. $\overrightarrow{ZJ} = \sqrt[3]{13.5} = 2.381101578ZJ$; $2\overrightarrow{ZC} = 1.25992105$

$\therefore \overrightarrow{ZC} = 2.381101578 \times 1.25992105 = 3$

$\because \overrightarrow{ZC}$ is the edge of the required cube, $\overrightarrow{ZC} = 3$

$\therefore \overrightarrow{ZC}$ Cube = $3^3 = 27$

$\because \overrightarrow{ZC}$ A cube with an edge is equal to, it is known that the times of the cube are equal to the cube,

$\therefore \overrightarrow{ZC}$ A cube with edges is equal to the times of a given cube.

7. Conclusion

Although the results of some examples show that the cubes made by the edges of the first, fourth, and fifth examples are several tens of billions of times different from the cubic ones, it is proved that if the cube roots of the edges of the given cubic cube are rational roots, the cubes made are equal to the times of the given volume. Conclusion: A cube made of a line segment equal to the edge of a given cube with a ruler is equal to the times of the given cube. It is proved that the ruler can make a cube equal to the times of a given cube, and the drawing method is to make a cube with a line segment equal to the edge of the known cube equal to the times of the known cube.

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