Cargo Operations of Express Air

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Abstract
This report focuses on solving the cargo transportation problem for Express Air, an air express company operating among three airports. The objective is to minimize costs while meeting the daily demand for cargo transportation between each origin-destination airport. A minimum cost optimization model is employed to find the optimal schedule. Sensibility checks are performed, and the results are analyzed, including various scenarios using the developed models. The mathematical model includes variables representing cargo amounts, repositioning aircraft, staying aircraft, and cargo left on the ground. The objective function aims to minimize the total cost of cargo left on the ground and repositioning aircraft. Constraints ensure the balance of aircraft and cargo flows and account for demand and cargo left from previous days. The optimal solution yields a total cost of 17,925, with repositioning aircraft accounting for 84.38% of the cost. Increasing the fleet size reduces costs, and the model can be applied to supply chain systems and online shopping logistics.

Keywords
Cargo scheduling, cost minimization, optimization model

1. Problem overview
In this paper, we intend to solve the problem of cargo transportation among three different airports for an air express company. Each day airplanes will carry cargo from one airport to another to satisfy the demand for the cargo. However, since the planes are limited, the cargo that needs transport cannot be at the destination on time, so we also need to consider delivering some cargo the next day and sometimes use empty planes from other places the day before to satisfy the need for cargo transportation.

And the transportation of empty planes costs money, and so does cargo left on the ground. For this problem, we need to schedule the cargo transportation to satisfy each airport's needs and minimize the cost of empty plane transportation and cargo cost left on the ground [1].

2. Data description
We need to arrange planes to fly through three airports: A, B, and C. And the origin-destination route will be A-B, A-C, B-A, B-C, C-A, C-B.

And the total number of aircraft we can arrange is 1200.

Amount of cargo (in aircraft loads) arriving into the system on each day that needs to be carried between each origin-destination airport (See above Table 1).

Cost of holding one full aircraft load of cargo on the ground: 10 per day

Cost of empty repositioning movements (See below Figure 1).

3. Optimization model overview
For this optimization model, we will use a minimum cost model and focus on minimizing the cost of empty plane transportation and cargo costs on the ground. And also, we would make sure there are enough planes to carry cargo...
every day for each route from origin-destination so that every day’s tasks can be performed. Besides, we need to guarantee that every day the number of planes will be less than the total number of planes. What’s more, we need to ensure that the aircraft at each airport the day before are more significant than the total number of planes that take off the next day from that airport [2].

Table 1. Amounts of cargo (in aircraft loads) arriving into the system on each day that needs to be carried between each origin-destination airport

<table>
<thead>
<tr>
<th>Origin-destination</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-B</td>
<td>100</td>
<td>200</td>
<td>100</td>
<td>400</td>
<td>300</td>
</tr>
<tr>
<td>A-C</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>B-A</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>B-C</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>C-A</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>C-B</td>
<td>400</td>
<td>200</td>
<td>300</td>
<td>200</td>
<td>400</td>
</tr>
</tbody>
</table>

Figure 1. Empty repositioning costs among different airports.

4. Mathematical detail

4.1 Variables:

Define:

- \( S \): A set of airports \( S = \{ A, B, C \} \)
- \( T \): day of the weekday \( T = \{ 1, 2, 3, 4, 5 \} \), 1 to 5 represent Monday to Friday
- \( X_{ijt} \): the amount of cargo in aircraft from \( i \) to \( j \) arriving into the system on \( t \) (\( \forall i \in S, \forall j \in S, i \neq j, \forall t \in T \))
- \( Y_{ijt} \): the amount of repositioning aircraft from \( i \) to \( j \) arriving on \( t \) (\( \forall i \in S, \forall j \in S, i \neq j, \forall t \in T \))
- \( Z_{it} \): the amount of aircraft staying at \( i \) on \( t \) (\( \forall i \in S, \forall t \in T \))
- \( L_{ijt} \): the amount of cargo supposed to deliver from \( i \) to \( j \) on \( t \) but left on the ground (\( \forall i \in S, \forall j \in S, i \neq j, \forall t \in T \))
- \( C \): cost of cargo on the ground one day, \( C = 10 \)
- \( D_{ijt} \): the amount of cargo (in aircraft loads) arriving into the system on \( t \) that need to be carried from \( i \) to \( j \) (\( \forall i \in S, \forall j \in S, i \neq j, \forall t \in T \)), the values of \( D_{ijt} \) are presented in above Table 1.
- \( E_{ij} \): the cost of repositioning from \( i \) to \( j \) (\( \forall i \in S, \forall j \in S, i \neq j \)), the values of \( E_{ij} \) are presented in above Figure 1

4.2 Objective Function

Minimize: \( \sum_{t} \sum_{j} \sum_{i} (C \cdot L_{ijt} + E_{ij} \cdot Y_{ijt}) \) (\( \forall i \in S, \forall j \in S, i \neq j, \forall t \in T \))

4.3 Constraints

Constraint 1: Total amount of all aircraft (cargo aircraft, repositioning aircraft, and staying aircraft) we can operate on the same day is 1200

\[
\sum_{i \in S} \sum_{j \in S} (X_{ijt} + Y_{ijt} + Z_{it}) = 1200 \quad (i \neq j, \forall t \in T)
\]

Constraint 2: The amount of aircraft left from or staying at the airport on \( t \) should equal the amount of aircraft arriv-
ing or staying at this airport on t-1
\[
\sum_{i \in S} (X_{ijt} + Y_{ijt}) + Z_{it} = \sum_{j \in S} (X_{ij(t-1)} + Y_{ij(t-1)}) + Z_{it} \quad (\forall i \in S, i \neq j, \forall t \in \{2, 3, 4, 5\}) \\
\sum_{i \in S} (X_{ij1} + Y_{ij1}) + Z_{i1} = \sum_{j \in S} (X_{ij5} + Y_{ij5}) + Z_{i5} \quad (\forall i \in S, i \neq j)
\]

Constraint 3: The amount of cargo from i to j into the system on t is not supposed to be greater than the amount of demand with the amount of cargo left that suppose to deliver from i to j on t-1
\[
X_{ijt} \leq D_{ijt} + L_{ij(t-1)} \quad (\forall i \in S, \forall j \in S, i \neq j, \forall t \in \{2, 3, 4, 5\}) \\
X_{ij1} \leq D_{ij1} + L_{ij5} - X_{ij1} \quad (\forall i \in S, \forall j \in S, i \neq j)
\]

Constraint 4: The amount of cargo every time left not less than the sum of the amount of demand and the amount of cargo left the day before with the amount of cargo that is taking out today
\[
L_{ijt} \geq D_{ijt} + L_{ij(t-1)} - X_{ijt} \quad (\forall i \in S, \forall j \in S, i \neq j, \forall t \in \{2, 3, 4, 5\}) \\
L_{ij1} \geq D_{ij1} + L_{ij5} - X_{ij1} \quad (\forall i \in S, \forall j \in S, i \neq j)
\]

4.4 Optimize

Optimal value: 17,925
Cost of repositioning aircraft: 15,125
Cost of cargo left on the ground: 2,800

Variables Details:

Table 2. Amount of cargo (in aircraft loads) exactly arriving into the system on each day between each origin-destination airport

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-B</td>
<td>290</td>
<td>200</td>
<td>100</td>
<td>400</td>
<td>110</td>
</tr>
<tr>
<td>A-C</td>
<td>30</td>
<td>70</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>B-A</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
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<tr>
<td>B-C</td>
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<td>25</td>
<td>25</td>
<td>25</td>
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<tr>
<td>C-A</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>C-B</td>
<td>330</td>
<td>270</td>
<td>300</td>
<td>200</td>
<td>400</td>
</tr>
</tbody>
</table>

Table 3. Amount of repositioning aircraft arriving into the system on each day between each origin-destination airport

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A-C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B-A</td>
<td>205</td>
<td>85</td>
<td>420</td>
<td>60</td>
<td>255</td>
</tr>
<tr>
<td>B-C</td>
<td>255</td>
<td>485</td>
<td>0</td>
<td>290</td>
<td>295</td>
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<tr>
<td>C-B</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4. Amount of cargo supposed to arrive into the system on each day between each origin-destination airport but left on the ground

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>190</td>
</tr>
<tr>
<td>A-C</td>
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<td>0</td>
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<tr>
<td>B-A</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>B-C</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>C-A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C-B</td>
<td>70</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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</tbody>
</table>

Table 5. Amount of aircraft staying at each airport on each day

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>35</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>240</td>
<td>75</td>
<td>0</td>
</tr>
</tbody>
</table>
5. Argument

Our schedule is sensible. First, we make sure the numbers of planes and cargo are sensible for transportation at each airport every day, we check the flow balance of the aircraft (including loaded aircraft, repositioning aircraft, and staying aircraft) each day, and we check the flow balance of the cargo at each airport on each day [3].

Besides, the weekly schedule is repeating since we used Monday as the next day of Friday in our model. So when we make sure every day from Monday to Friday, then Friday to Monday, the number of planes and cargo of each origin-destination schedule is sensible, then this routine can be performed.

What's more, we have all the constraints, so the optimal value that is found by this model is acceptable, including all cost components described in the project.

6. Analysis of Result and finding

6.1 The analysis of result

In this model, we found the optimal value is 17,925, including 15,125 on the cost of repositioning aircraft and 2,800 on the cost of cargo left on the ground. Based on the result, the cost of repositioning aircraft took around 15,125/17,925 = 84.38% of the total cost [4].

6.2 The benefits of increasing the fleet size

The total cost of this model will be reduced by increasing the fleet size at the beginning. After we increase 190 aircraft, the total cost will reach the minimum cost, which is 15125 (Graph 1). Therefore, we can get a better optimal value if we have more available aircraft until we reach the minimum cost of 15125 in this model.

6.3 The effects of having more cargo to carry over different origin-destination pairs

In this experiment, we tested the different increasing sizes of cargo on some selected origin-destination pairs. From airport A to B on Monday:

In this pair, we found that the total cost increases by increasing the size of the cargo until it reaches the infeasible size, which is 276.

Figure 1. Cost change by increasing fleet size.

Figure 2. Cost with the size of cargo from airport A to B on Monday.
From airport B to C on Wednesday:
In this pair, we found that the cost will decrease at first when the cargo from airport B to C on Wednesday increases. However, after it is larger than 260, the cost will also increase as the cargo need increases. Comparing this graph and the graph of the difference from airport A to B on Monday, we found that if the cargo demand is small on that origin-destination routine on that day, the company can reduce some costs by increasing that cargo demand.

![Graph](image)

**Figure 3. Cost with the size of cargo from airport B to C on Wednesday.**

7. Application

This model, besides being used in aircraft and cargo movement cycles, can be used in some supply chain systems, such as the food supply in restaurants. A restaurant food supplier supplies some restaurants with food weekly or monthly. They have to organize the routine and schedule. What’s more, this model should be adaptive to some special cases, such as the amount of supply change by holiday. Also, this model can be used in online shopping systems. We need to minimize costs while we need to make sure all products are shipped from product providers and customers, which is also similar to this problem [5].

References


