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A Laplace Adomian Decomposition Method for Fractional Order Infection Model

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Abstract

Arbitrary order calculus to model real phenomena has been applied to various fields such as physics, chemisty, biology, etc. Therefore, more and more researchers prefer to use fractional order to describe infectious disease models. This article mainly discusses the dynamics of one susceptible-exposed-infected-recovered (SEIR) model of fractional order in Caputo sense. The premise of this model is that there is no vaccination. Recovered person will lose immunity and then return to susceptible group after a period of time. The approximate solution will be obtained with Laplace Adomian decomposition method (LADM) which has been proved to be an effective and reliable approach. Approximate results can be obtained through fewer iterations, which shows the effectiveness and simplicity of the LADM. From the graphical results it is suggested the flexibility and practicability of fractional differential. It is also indicated that universal vaccination in time is essential, otherwise the number of infected people is likely to continue to increase for a long time.

Keywords

SEIR model, LADM, Fractional derivatives, Numerical solutions

1. Introduction

Since the outbreak of COVID-19 epidemic in 2019, there has been another wave of research on infectious disease models. Ian Cooper developed a susceptible-infected-recovered (SIR) model about the novel COVID-19 disease to investigate its spread within a community in 2020 [1]. S He et al. [2] proposed a SEIR model to simulate the process of COVID-19 in 2020. CY Yang defined a model based on the traditional SIR model according to the COVID-19 situation in Wuhan, China [3]. Y Yuan investigated a model for transmission of the COVID-19 that considers personal protection awareness in 2022 and was committed to designing the optimal control strategies [4].

In the last few decades, scientists have successfully used arbitrary order calculus to model real phenomena. It has been applied to various fields such as physics, chemisty, biology, etc. Therefore, more and more researchers prefer to use fractional order to describe infectious disease models. Area et al. [5] discussed the fractional order SEIR Ebola model in terms of the Riemann-Liouville fractional order derivative. R Almeida et al. [6] presented an epidemiological model involving the Caputo fractional derivative and computed the basic reproduction number. To explore the effects of isolation, D.Baleanu et al. [7] generalized a model using the general concept of fractional order introduced by Y.Luchko and M.Yamamoto [8]. A.Badr analyzed a new vaccinated SARS-CoV-2 epidemic model under fractional differentiation with respect to the Caputo operator [9]. A fractional order model in Caputo sense with the power law kernel was outlined to predict future behavioral trends in the number of confirmed cases and deaths in the Indian COVID-19 outbreak [10]. In addition, some authors considered an epidemic model with Atangana-Baleaue fractional derivative and solved the approximate solution by decomposition method [11]. Some researchers [12] analyzed the susceptible-vaccinated-infective model in order to explore the effect of vaccination on the spread of the epidemic in India.

There are many numerical methods that can be used to find the approximate solutions of these mathematical models, such as homotopy analysis method (HAM), differential transformed method (DTM). Adomian decomposition method (ADM) was proposed by G.Adomian [13]. One of the greatest advantages of the Adomian decomposition method is that it can be used to solve all types of integral and differential equations. It is a universal method which considers the approximate solution of a non-linear equation as an infinite series which usually converges to the exact solution. The LADM is to perform Laplace transform on the differential equations before using ADM [14-16]. It is to be noted that LADM is more powerful than standard ADM. Inspired by the above-mentioned literatures, the main work of this paper is to describe the classcial model ignoring vaccination therein [17] with fractional order and then to find the solution by LADM.

This paper consists of five sections. Some preliminaries on the model to be discussed in this article and fractional derivative are presented in part 2. The whole process of solving such problem with LADM is explained in detail in part 3. In part 4, a specific example is solved with the technique to receive the numerical results. The last part is a summary of the full paper.

2. Preliminaries

This part introduces some important knowledge required about fractional calculus. And the classical SEIR model is expanded with the help of Caputo fractional-order operator.

2.1 Classical SEIR model

Using [17], the authors proposed the following susceptible-exposed-infected-recovered (SEIR) model

$$\begin{cases} S'(t) & = A - uS(t) - S(t)I(t)\beta + wR(t), \\ E'(t) & = S(t)I(t)\beta - (\alpha + u)E(t), \\ I'(t) & = E(t)\alpha - (\gamma + u + m)I(t), \\ R'(t) & = \gamma I(t) - (u + w)R(t), \end{cases}$$

with given the initial conditions $S(0) = s_0, E(0) = e_0, I(0) = i_0, R(0) = r_0$.

Where$S(t), E(t), I(t)$ and$R(t)$ are the number of susceptible, exposed, infected, and removed individuals at time t. The difference between this model and classical SIR model is that one more exposed group is divided. When people come into contact with the infected, the symptoms will not show immediately. So this group is infections but not quarantined during the incubation period. The parameters $A, u, \beta, \alpha, \gamma, m, w$ represent the recruitment rate, the natural death rate, the rate at which susceptible people develop into latent, the rate at which latent develop into patients, patient recovery rate, mortality due to infection, rate of losing immunity, respectively.

2.2 Fractional SEIR model

Definition2.1. *[18]* Let $\alpha \in [0,1]$, $f \in H^1(a, b)$, the usual Caputo fractional time derivative of order α, given by

$${}_{\alpha}^{c}Df(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f'(s)}{(t-s)^{\alpha}} ds .$$

Definition2.2. *[19]* Given a function $f(t)$ defined for $0 < t < \infty$, the following integral converges in a certain region of s, where s is a complex parameter.

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st} dt .$$

Thus the function $F(s)$ is the Laplace transform of $f(t)$. $\mathcal{L}$ is Laplace operator.

The operation of the original function is the inverse Laplace transform, the formula is

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j} \int_{m-j\infty}^{m+j\infty} F(s)e^{-st} ds .$$

Proposition2.1. *[18,20]* Let $0 < \alpha < 1$, then the Laplace transform of Caputo derivative is given by

$$\mathcal{L}\{{}_{\alpha}^{c}Df(t)\} = s^{\alpha}\mathcal{L}\{f(t)\} - s^{\alpha-1}f(0).$$

Since fractional calculus is very suitable for characterizing materials and processes with memory and genetic properties, we proposed the fractional order SEIR model as followers

$$\begin{cases} {}_{p}^{c}DS(t) & = A - uS(t) - S(t)I(t)\beta + wR(t), \\ {}_{p}^{c}DE(t) & = S(t)I(t)\beta - (\alpha + u)E(t), \\ {}_{p}^{c}DI(t) & = E(t)\alpha - (\gamma + u + m)I(t), \\ {}_{p}^{c}DR(t) & = \gamma I(t) - (u + w)R(t), \end{cases} \tag{2.1}$$

with given the initial conditions $S(0) = s_0, E(0) = e_0, I(0) = i_0, R(0) = r_0$, where ${}_{p}^{c}D$ is the Caputo operator and $p \in [0,1]$ suggests fractional time derivative.

3. Solution procedure

The present part is devoted to expound how to use the LADM to solve equation (2.1). Applying laplace transform on both sides of the model (2.1), we get

$$\begin{cases} \mathcal{L}\{{}_{p}^{c}DS(t)\} & = \mathcal{L}\{A - uS(t) - S(t)I(t)\beta + wR(t)\}, \\ \mathcal{L}\{{}_{p}^{c}DE(t)\} & = \mathcal{L}\{S(t)I(t)\beta - (\alpha + u)E(t)\}, \\ \mathcal{L}\{{}_{p}^{c}DI(t)\} & = \mathcal{L}\{E(t)\alpha - (\gamma + u + m)I(t)\}, \\ \mathcal{L}\{{}_{p}^{c}DR(t)\} & = \mathcal{L}\{\gamma I(t) - (u + w)R(t)\}. \end{cases} \tag{3.1}$$

Therefore we have

$$\begin{cases} s^p \mathcal{L}\{S(t)\} - s^{p-1}S(0) & = \frac{A}{s} - \mathcal{L}\{uS(t) + S(t)I(t)\beta - wR(t)\}, \\ s^p \mathcal{L}\{E(t)\} - s^{p-1}E(0) & = \mathcal{L}\{S(t)I(t)\beta - (\alpha + u)E(t)\}, \\ s^p \mathcal{L}\{I(t)\} - s^{p-1}I(0) & = \mathcal{L}\{E(t)\alpha - (\gamma + u + m)I(t)\}, \\ s^p \mathcal{L}\{R(t)\} - s^{p-1}R(0) & = \mathcal{L}\{\gamma I(t) - (u + w)R(t)\}. \end{cases}$$

Substituting the initial conditions into the above equations, then we have

$$\begin{cases} \mathcal{L}\{S(t)\} & = \frac{s_0}{s} + \frac{A}{s^{p+1}} - \frac{1}{s^p}\mathcal{L}\{uS(t) + S(t)I(t)\beta - wR(t)\}, \\ \mathcal{L}\{E(t)\} & = \frac{e_0}{s} + \frac{1}{s^p}\mathcal{L}\{S(t)I(t)\beta - (\alpha + u)E(t)\}, \\ \mathcal{L}\{I(t)\} & = \frac{i_0}{s} + \frac{1}{s^p}\mathcal{L}\{E(t)\alpha - (\gamma + u + m)I(t)\}, \\ \mathcal{L}\{R(t)\} & = \frac{r_0}{s} + \frac{1}{s^p}\mathcal{L}\{\gamma I(t) - (w + u)R(t)\}. \end{cases} \tag{3.2}$$

The solutions are usually expressed in the form of infinite series as follows

$$S(t) = \sum_{n=0}^{\infty} S_n(t), E(t) = \sum_{n=0}^{\infty} E_n(t), I(t) = \sum_{n=0}^{\infty} I_n(t), R(t) = \sum_{n=0}^{\infty} R_n(t).$$

S(t)I(t) is the nonliner term in the model and it can be decomposed by Adomian polynomial in the form of $S(t)I(t) = \sum_{n=0}^{\infty} A_n(t)$.$A_n$ is the so-called Adomian polynomial.Its convergence has been discussed in [21, 22] and

$$A_n(t) = \frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\left[\sum_{k=0}^{n} \lambda^k S_k(t) \sum_{k=o}^{n} \lambda^k I_k(t)\right]_{\lambda=0}.$$

We can calculate the first four terms

$$A_0 = S_0I_0, A_1 = S_0I_1 + S_1I_0, A_2 = S_0I_2 + S_1I_1 + S_2I_0, A_3 = S_0I_3 + S_1I_2 + S_2I_1 + S_3I_0 \dots \dots$$

Substituting the above infinite series form into equation (3.2), we have

$$\begin{cases} \mathcal{L}\{\sum_{n=0}^{\infty} S_n(t)\} & = \frac{s_0}{s} + \frac{A}{s^{p+1}} - \frac{1}{s^p}\mathcal{L}\{u\sum_{n=0}^{\infty} S_n(t) + \beta\sum_{n=0}^{\infty} A_n(t) - w\sum_{n=0}^{\infty} R_n(t)\}, \\ \mathcal{L}\{\sum_{n=0}^{\infty} E_n(t)\} & = \frac{e_0}{s} + \frac{1}{s^p}\mathcal{L}\{\beta\sum_{n=0}^{\infty} A_n(t) - (\alpha + u)\sum_{n=0}^{\infty} E_n(t)\}, \\ \mathcal{L}\{\sum_{n=0}^{\infty} I_n(t)\} & = \frac{i_0}{s} + \frac{1}{s^p}\mathcal{L}\{\alpha\sum_{n=0}^{\infty} E_n(t) - (\gamma + u + m)\sum_{n=0}^{\infty} I_n(t)\}, \\ \mathcal{L}\{\sum_{n=0}^{\infty} R_n(t)\} & = \frac{r_0}{s} + \frac{1}{s^p}\mathcal{L}\{\gamma\sum_{n=0}^{\infty} I_n(t) - (w + u)\sum_{n=0}^{\infty} R_n(t)\}. \end{cases} \tag{3.2}$$

By matching the terms on both sides of the equation (3.3), we can get the following iterative algorithm

$$\begin{cases} \mathcal{L}\{S_0(t)\} &= \frac{s_0}{s} + \frac{A}{s^{p+1}}, \\ \mathcal{L}\{S_1(t)\} &= -\frac{1}{s^p}\mathcal{L}\{uS_0(t) + A_0(t)\beta - wR_0(t)\}, \\ \mathcal{L}\{S_2(t)\} &= -\frac{1}{s^p}\mathcal{L}\{uS_1(t) + A_1(t)\beta - wR_1(t)\}, \\ & \dots \\ \mathcal{L}\{S_{n+1}(t)\} &= -\frac{1}{s^p}\mathcal{L}\{uS_n(t) + A_n(t)\beta - wR_n(t)\}, \end{cases}, \quad \begin{cases} \mathcal{L}\{E_0(t)\} &= \frac{e_0}{s}, \\ \mathcal{L}\{E_1(t)\} &= \frac{1}{s^p}\mathcal{L}\{A_0(t)\beta - (\alpha+u)E_0(t)\}, \\ \mathcal{L}\{E_2(t)\} &= \frac{1}{s^p}\mathcal{L}\{A_1(t)\beta - (\alpha+u)E_1(t)\}, \\ & \dots \\ \mathcal{L}\{E_{n+1}(t)\} &= \frac{1}{s^p}\mathcal{L}\{A_n(t)\beta - (\alpha+u)E_n(t)\}, \end{cases}$$

$$\begin{cases} \mathcal{L}\{I_0(t)\} &= \frac{i_0}{s}, \\ \mathcal{L}\{I_1(t)\} &= \frac{1}{s^p}\mathcal{L}\{\alpha E_0(t) - (\gamma+u+m)I_0(t)\}, \\ \mathcal{L}\{I_2(t)\} &= \frac{1}{s^p}\mathcal{L}\{\alpha E_1(t) - (\gamma+u+m)I_1(t)\}, \\ & \dots \\ \mathcal{L}\{I_{n+1}(t)\} &= \frac{1}{s^p}\mathcal{L}\{\alpha E_n(t) - (\gamma+u+m)I_n(t)\}, \end{cases}, \quad \begin{cases} \mathcal{L}\{R_0(t)\} &= \frac{r_0}{s}, \\ \mathcal{L}\{R_1(t)\} &= \frac{1}{s^p}\mathcal{L}\{\gamma I_0(t) - (w+u)R_0(t)\}, \\ \mathcal{L}\{R_2(t)\} &= \frac{1}{s^p}\mathcal{L}\{\gamma I_1(t) - (w+u)R_1(t)\}, \\ & \dots \\ \mathcal{L}\{R_{n+1}(t)\} &= \frac{1}{s^p}\mathcal{L}\{\gamma I_n(t) - (w+u)R_n(t)\}. \end{cases}$$

Subsequently using the Laplace inverse transformation to the first three items of each equation, eventually we can get

$$\begin{cases} S_0(t) &= s_0 + A\frac{t^p}{\Gamma(p+1)}, \\ S_1(t) &= -X_0\frac{t^p}{\Gamma(p+1)} - A(u+\beta i_0)\frac{t^{2p}}{\Gamma(2p+1)}, \\ S_2(t) &= [(u+i_0\beta)X_0 - s_0\beta Y_0 + wZ_0]\frac{t^{2p}}{\Gamma(2p+1)} \\ & +A\left[(u+i_0\beta)^2 + Y_0\beta\frac{\Gamma(2p+1)}{\Gamma^2(p+1)}\right]\frac{t^{3p}}{\Gamma(3p+1)}, \\ & \dots \end{cases}, \quad \begin{cases} E_0(t) &= e_0, \\ E_1(t) &= G_0\frac{t^p}{\Gamma(p+1)} + Ai_0\beta\frac{t^{2p}}{\Gamma(2p+1)}, \\ E_2(t) &= [s_0\beta Y_0 - i_0\beta X_0 - (\alpha+u)G_0]\frac{t^{2p}}{\Gamma(2p+1)} \\ & +A\beta\left[Y_0\frac{\Gamma(2p+1)}{\Gamma^2(p+1)} - i_0(u+i_0\beta) - i_0(\alpha+u)\right]\frac{t^{3p}}{\Gamma(3p+1)}, \\ & \dots \end{cases}$$

$$\begin{cases} I_0(t) &= i_0, \\ I_1(t) &= Y_0\frac{t^p}{\Gamma(p+1)}, \\ I_2(t) &= [\alpha G_0 - (\gamma+u+m)Y_0]\frac{t^{2p}}{\Gamma(2p+1)} + Ai_0\beta\alpha\frac{t^{3p}}{\Gamma(3p+1)} \\ & \dots \end{cases}, \quad \begin{cases} R_0(t) &= r_0, \\ R_1(t) &= Z_0\frac{t^p}{\Gamma(p+1)}, \\ R_2(t) &= [\gamma Y_0 - (w+u)Z_0]\frac{t^{2p}}{\Gamma(2p+1)}, \\ & \dots \end{cases}$$

where

$$\begin{aligned} X_0 &= us_0 + i_0 s_0\beta - wr_0, \\ Y_0 &= e_0\alpha - i_0(\gamma+u+m), \\ Z_0 &= i_0\gamma - (u+w)r_0, \\ G_0 &= s_0 i_0\beta - (\alpha+u)e_0. \end{aligned}$$

By analogy, we can compute the remaining terms in the same way.

4. Numerical results

In this part, we will assign values to all parameters and obtain the results from our work. Let's take $s_0 = 999, e_0 = 1, i_0 = 0, r_0 = 0, A = 1000/76, u = 1/76, \beta = 0.21/1000, \alpha = 1/7, \gamma = 1/14, m = 0$. Previous equations can be written as

$$\begin{aligned} S_0 &= 999 + \frac{1000t^p}{76\Gamma(p+1)}, E_0 = 1, I_0 = 0, R_0 = 0; \\ S_1 &= -\frac{999t^p}{76\Gamma(p+1)} - \frac{1000t^{2p}}{76^2\Gamma(2p+1)}, E_1 = -\frac{83t^p}{532\Gamma(p+1)}, \\ I_1 &= \frac{t^p}{7\Gamma(p+1)}, R_1 = 0; \\ S_2 &= \frac{999t^{2p}}{76^2\Gamma(2p+1)} - \frac{2997t^{2p}}{100000\Gamma(2p+1)} - \frac{\Gamma(2p+1)}{\Gamma^2(p+1)}\frac{21t^{3p}}{53200\Gamma(3p+1)} + \frac{1000t^{3p}}{76^3\Gamma(3p+1)}, \\ E_2 &= \frac{2997t^{2p}}{100000\Gamma(2p+1)} + \frac{\Gamma(2p+1)}{\Gamma^2(p+1)}\frac{21t^{3p}}{7600\Gamma(3p+1)} + \frac{83^2t^{2p}}{532^2\Gamma(2p+1)}, \\ I_2 &= -\frac{128t^{2p}}{3724\Gamma(2p+1)}, R_2 = \frac{t^{2p}}{98\Gamma(2p+1)}. \end{aligned}$$

Therefore, after three terms the solutions are given by

$$
\begin{aligned}
\widetilde{S(t)} &= 999 + \frac{t^p}{76\Gamma(p+1)} - \left(\frac{1000}{76^2} + \frac{2997}{100000}\right)\frac{t^{2p}}{\Gamma(2p+1)} + \left(\frac{1000}{76^3} - \frac{21\Gamma(2p+1)}{53200\Gamma^2(p+1)}\right)\frac{t^{3p}}{\Gamma(3p+1)},\\
\widetilde{E(t)} &= 1 - \frac{83t^p}{532\Gamma(p+1)} + \left(\frac{2997}{100000} + \frac{83^2}{532^2}\right)\frac{t^{2p}}{\Gamma(2p+1)} + \frac{\Gamma(2p+1)}{\Gamma^2(p+1)}\frac{21t^{3p}}{7600\Gamma(3p+1)},\\
\widetilde{I(t)} &= \frac{t^p}{7\Gamma(p+1)} - \frac{128t^{2p}}{3724\Gamma(2p+1)},\\
\widetilde{R(t)} &= \frac{t^{2p}}{98\Gamma(2p+1)}.
\end{aligned}
$$

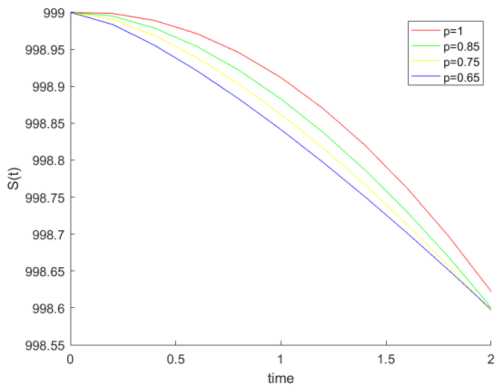

Figure 1. The numerical result about susceptible at different order.

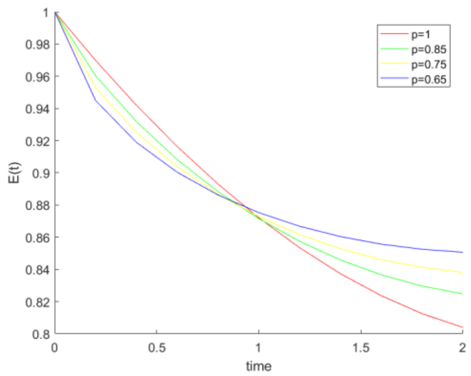

Figure 2. The numerical result about exposed at different order.

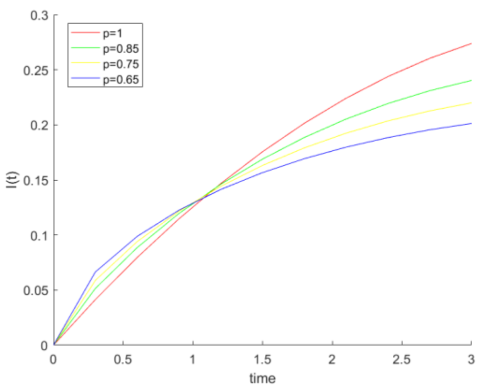

Figure 3. The numerical result about infected at different order.

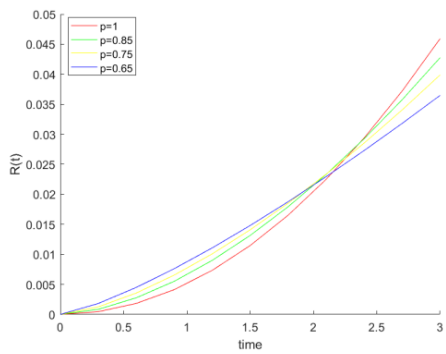

Figure 4. The numerical result about recovered at different order.

From the above figures, we can observe that with the reduction of the order of the fractional derivative from 1 to 0.65, the number of susceptible people began to decline earlier and earlier. As the time order decreases, the recovery rate at the same moment becomes slower. The graphs above also state that the number of susceptible people dropped very slowly without vaccination and the number of infections continues to increase over a long period of time.

5. Conclusion

In this paper, we have solved the SEIR model under different fractional order in sense of Caputo fractional operator by using Laplace Adomian decomposition method. Approximate results can be obtained through fewer iterations, which shows the effectiveness and simplicity of the LADM. From the graphical results it is suggested the flexibility and practicability of fractional differential. It is also indicated that universal vaccination in time is essential, otherwise the number of infected people is likely to continue to increase for a long time.

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