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A Stochastic Analysis of Stock Market Price Fluctuations for Capital Market

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Abstract

This paper studied the stochastic analysis of stock market expected returns for investors. The detailed conditions for obtaining the drifts, volatilities and variances of four different stocks were considered. We compared the variances of four different stocks using our criteria for selection and the result showed that stock (1) is the best among the stocks of different companies, which is in agreement with the work of Ofomata et al. From the stochastic analysis of the model, systems of non-linear ordinary differential equations were developed by means of covariance matrix on the stochastic part of the expected returns of investors while the deterministic part a function of the drift parameter as a column vector. We imposed a constraint that led the drift parameter to zero and working with the stochastic part of the equation by exploring correlation matrix solution. This way, a statistical goodness of fit was proposed.

Keywords

Stock market price, drift, volatility, variance, SDE and Stochastic analysis

1. Introduction

The stock price fluctuations has kept our economy been unstable; investors, families and federal Government are no-longer planning their future due to uncertainty involve in stock trading. Stock fluctuations results to panic buying of goods and services; which put fear in the lives of the masses. Even financial analysts who invest in stock market are usually not conscious of the stock market behavior. They also went through this problem of stock trading; without knowing which stocks to buy and sell in order to maximize profits. Both financial analysts and potential investors need regular information in forecasting the behavior of stock prices in Nigeria for the betterment of our economy [1].

So because of these great challenges, researchers have taken it into consideration to study the characteristics of the stock price fluctuations for capital market. Therefore, studying stock behavior is essential because it the medium in which the physical quantities are modeled for the purpose of practical findings. That is why, it is important to understand the dynamic nature of the physical problem to be solved for proper mathematical prediction. Hence, mathematical models grow out of equations that verify how a system changes from one state to the other [2].

Nevertheless, the price evolution of a risky assets are usually modeled as the trajectory of a risky assets that are usually of a diffusion process defined on some underlying probability space ,with the geometric Brownian motion the paramount tool used as the established reference model [3].

Modeling of Stock market prices cannot be over emphasized due to its numerous applications in the fast growing field of science and technology. For instance, [4] considered the unstable nature of stock market forces using proposed differential equation model. In the work of [5] studied stability analysis of stochastic model of price change at the floor of a stock market. In their research, precise conditions are obtained which determines the equilibrium price and growth rate of stock shares.

[6] considered stochastic analysis of the behavior of stock prices. Results reveal that the proposed model is efficient for the prediction of stock prices. In the same vain, [7] studied the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in their research the drift and volatility coefficients for the stochastic differential equations were determined and the Euler-Maruyama method for system of SDE'S was used to stimulate the stock prices [8], built the geometric Brownian Motion and studied the accuracy of the model with detailed analysis of simulated data.

More so, [3] worked on stochastic model of the fluctuation of stock market price is considered. Here conditions for determining the equilibrium price, sufficient conditions for dynamic stability and convergence to equilibrium of the growth rate of the value function of shares. On the other hand, [9] considered a stochastic model of price changes at the floor of stock market. In their research, the equilibrium price and the market growth rate of shares were determined. See [10] for considerable extensions and constrains subsequently in this particular area of study.

More importantly, the aim of this paper is first, to present a stochastic analysis of stock market price fluctuations for capital market which aimed at determining the drifts, volatilities and variances then using a certain criteria to select the best model, from the stochastic analysis we considered the two parameter of the model and imposed a conditions which the stock drift was led to zero; allowing us to solve on the volatility part and goodness of fit test was obtained and significantly reliable. To the best of our knowledge, these novel contributions have not been seen elsewhere in this dynamic area of financial mathematics.

This paper is arranged as follows: Section 2 presents the methods, the problem formulation is seen in Subsection 2.1, Subsection 2.2 is parameter Estimation, and goodness of fit test for equal stock price correlation is seen in Subsection 2.3, Results are seen in Section 3, while the discussion is presented in Section 4 and paper is concluded in Section 5.

2. Methods

Let $S(t)$ be the price of some risky asset at time t ,and μ ,an expected rate of returns on the stock and dt as a relative change during the trading days such that the stock price follows a random walk which is governed by a stochastic differential equation.

$$dS(t) = \mu S(t)dt + \sigma S(t)dW_t \tag{1}$$

Where, μ is drift and σ the volatility of the stock, W_t is a Brownian motion or Wiener's process on a probability space $(\Omega, \xi, \wp), \xi$ is a $\sigma-$algebra generated by $W_t, t \geq 0$.

Definition 1.1: A standard Brownian Motion is simply a stochastic process $\{B_t\}_{t\in\tau}$ with the following properties:

1) With probability 1, $B_0 = 0$.
2) For all $0 \leq t_1 \leq t_2 \leq \cdots \leq t_n$, the increments $B_{t2-}B_{t1}, B_{t3-}B_{t2}, B_{t2}, \cdots, B_{tn} - B_{tn-1}$ are independent.
3) For $t \geq s \geq 0,\ B_t - B_s \sim N(0, t-s)$.
4) With probability 1, the function $\rightarrow B_i$ is continuous.

Stock Price Modelling

Theorem 1.1: (Ito's formula) Let $(\Omega, \beta, \mu, \mathrm{F}(\beta))$ be a filtered probability space $X = \{X, t \geq 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, \mathrm{F}(\beta))$ processing a quadratic variation (X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t) \tag{2}$$

$t \in \Re$ and for $u = u(t, X(t) \in C^{1\times 2}(\Pi \times \mathbb{R}))$

$$du(t, X(t)) = \left\{\frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2}f^2\frac{\partial^2 u}{\partial x^2}\right\}d\tau + f\frac{\partial u}{\partial x}dW(t) \tag{3}$$

Using theorem 1.1 and equation (3) comfortably solves the SDE with a solution given below:

$$S(t) = S_0\ \exp\left\{\sigma dW(t) + \left(\alpha - \frac{1}{2}\sigma^2\right)t\right\}, \forall t \in [0,1]$$

Following the properties of standard Brownian motion process for $n \geq 1$ such that any sequence time has it thus: $0 \leq t_0 \leq t_1 \leq \cdots \leq t_n$, hence, we have Euler's method for discretization of the SDE as follows:

$$\ln S_t - \ln S_{t-1} = \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\left(W_t - W_{t-1}\right) \tag{4}$$

Definition 1.2: A random variables say $W_t - W_{t-1}$ are functions, σ and therefore independent; which has a standard normal distribution with zero mean and variance one respectively.

From (4) if we let $y = \ln S_t - \ln S_{t-1}$, $\varepsilon = W_t - W_{t-1}$ and $\Delta t = 1$ (4) becomes

$$y_t = \mu - \frac{1}{2}\sigma^2 + \sigma\varepsilon_t \tag{5}$$

Linking (4) and (5) gives:

$$\ln S_t = \ln S_{t-1} + \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon(\sqrt{dt}) \tag{6}$$

Divide both sides by ln gives

$$S_t = S_{t-1} e^{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon(\sqrt{dt})} \tag{7}$$

2.1 Problem Formulation

Let S_1, S_2, S_3 and S_4 represents daily prices in naira of four selected stocks. Time t is counted for trading days in multiples of fundamental unit, say days. Also, let an $N \times n$ data matrix associated with $S_0(1), S_0(2), S_0(3)$ *and* $S_0(4)$ be X_{it}, we consider N stocks over n trading days; time horizon. For each of four X_i, we define the vector D_{it} as follows:

$$D1_{it} = \frac{1}{N}\sum_{i=1}^{N}\left(D_{i1}, D_{i2}, D_{i3}, \cdots, D_{in}\right) \tag{8}$$

$$D2_{it} = \frac{1}{N}\sum_{i=1}^{N}\left(D_{i1}, D_{i2}, D_{i3}, \cdots, D_{in}\right) \tag{9}$$

$$D3_{it} = \frac{1}{N}\sum_{i=1}^{N}\left(D_{i1}, D_{i2}, D_{i3}, \cdots, D_{in}\right) \tag{10}$$

$$D4_{it} = \frac{1}{N}\sum_{i=1}^{N}\left(D_{i1}, D_{i2}, D_{i3}, \cdots, D_{in}\right) \tag{11}$$

Thus, from where further statistics are derived the best stock is chosen in terms of minimum variance criterion, $\min\left(\text{var}(D_{it}), i = 1, \cdots, N\right)$; for non-zero element only.

Following the method of [7], we define covariance matrix as:

$$B(\text{t}, \text{S}_1, \text{S}_2, \text{S}_3, \text{S}_4) = \begin{pmatrix} dS_1 & dS_1S_2 & dS_1S_3 & dS_1S_4 \\ dS_2S_1 & dS_2 & dS_2S_3 & dS_2S_4 \\ dS_3S_1 & dS_3S_2 & dS_3 & dS_3S_4 \\ dS_4S_1 & dS_4S_2 & dS_4S_3 & dS_4 \end{pmatrix}^{\frac{1}{2}} \tag{12}$$

Where covariance matrix represents the volatility coefficient of the stochastic differential equation. It is sufficient to know that covariance matrix is a positive definite symmetric matrix with a positive definite square-root. While stock drift is represented in vector form below:

$$\mu(t, S_1, S_2, S_3, S_4) = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} \tag{13}$$

Combining (9) and (10) gives

$$dS(t) = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} dt + \begin{pmatrix} dS_1 & dS_1S_2 & dS_1S_3 & dS_1S_4 \\ dS_2S_1 & dS_2 & dS_2S_3 & dS_2S_3 \\ dS_3S_1 & dS_3S_2 & dS_3 & dS_3S_4 \\ dS_4S_1 & dS_4S_2 & dS_4S_3 & dS_4 \end{pmatrix}^{\frac{1}{2}} dW(t) \tag{14}$$

Which invariably gives us system of stochastic differential equations .Setting $dt = 0$, gives a stocks correlation matrix solution structure.

$$R = \begin{pmatrix} dS_1 & dS_1S_2 & dS_1S_3 & dS_1S_4 \\ dS_2S_1 & dS_2 & dS_2S_3 & dS_2S_3 \\ dS_3S_1 & dS_3S_2 & dS_3 & dS_3S_4 \\ dS_4S_1 & dS_4S_2 & dS_4S_3 & dS_4 \end{pmatrix} \tag{15}$$

2.2 Parameter Estimation

Using data provided in [7], we estimate the following: volatility (σ) and the drift (μ) of the stock price for the four selected company such as S1-INTERBREW, S2-AP, S3-ASHAKACEM and S4-STANBIC. The daily stock prices is made of 60 observations where we partitioned each stock to be four (4) compartments which gave fifteen (15) Tables.

The formular for the volatility and drift of the stock price is defined as follows:

2.2.1 the volatility, σ : let S_i represent each of the initial stock prices at the end of $i-th$ trading period, $T = t, -t_{i-1}, i \geq 1$. We define it as the logarithm of the daily return on each of four compartments of stock prices such that:

$$\mu_i = \ln\left[\frac{S_i}{S_{i-1}}\right] \tag{16}$$

The mean of each of stock prices are given as

$$\bar{u} = \frac{1}{N}\sum_{i=1}^{n} u_i \tag{17}$$

The standard deviation is given as

$$v = \frac{\sqrt{\sum |u_i - \bar{u}|^2}}{n-1} \tag{18}$$

The volatility of the daily stock return:

$$\sigma = \frac{v}{\sqrt{\tau}} \tag{19}$$

2.2.2 the Drift Parameter, μ : this simply means expected annual rate of return which is given as

$$\mu = \frac{\bar{u}}{\tau} + \frac{1}{2}\sigma^2 \tag{20}$$

2.3 Goodness of fit test for equal stock price Correlation structure

To test for this structure, let

$$H_0 : \rho = \rho_0$$
$$H_0 : \rho \neq \rho_0$$

Lawley's procedure requires the stock price quantities

$$\bar{r}_k = \frac{1}{\rho - 1}\sum_{i=1}^{\rho} r_{ik}, k = 1, 2, \cdots, \rho, \bar{r} = \frac{2}{\rho - 1}\sum_{1<}\sum_{k} r_{ik}$$

$$\hat{r} = \frac{(\rho-1)^2\left\{1-(1-\bar{r})^2\right\}}{\rho-(\rho-2)(1-\bar{r})^2} \tag{21}$$

Where $\bar{r}_k$ is the average of the off-diagonal elements in the kth column or row of k, $\bar{r}$ is the general average of the off-diagonal elements.

Decision rule: Reject H_0 in favour of H_1 if

$$T = \frac{(\mathrm{n}-1)}{(1-\bar{r})^2}\left\{\sum_{1<}\sum_{k}(r_{ik}-\bar{r})^2 - \hat{r}\sum_{k=1}^{\rho}(\bar{r}_k-\bar{r})^2\right\} > \chi^2_{(\rho+1)(\rho-2)/2\,(\alpha)} \tag{22}$$

Where $\chi^2_{(\rho+1)(\rho-2)/2\,(\alpha)}$ is the upper $(100\alpha)th$ percentile of a chi-square distribution with $(\rho+1)(\rho-2)/2$ degree of freedom.

3. Results

This Section presents the computational results for the problem stated in Section 2 and Subsections. The graphical results are implemented using Matlab programming language.

Using equation (14) gives a system of stochastic differential equation

$$dS(t) = \begin{pmatrix} 0.7843 \\ 0.1294 \\ 0.4654 \\ 6.5328 \end{pmatrix} dt + \begin{pmatrix} 0.2486 & 0.1122 & 0.1503 & 0.0624 \\ 0.1122 & 0.1039 & 0.0616 & 0.0849 \\ 0.1503 & 0.0616 & 0.0894 & 0.1323 \\ 0.0624 & 0.0849 & 0.1323 & 0.37 \end{pmatrix} dW(\mathrm{t})$$

This gives systems of non-linear stochastic differential equation below:

$$dS_1 = 0.7843dt + 0.2486dW_1 + 0.1122dW_2 + 0.1503dW_3 + 0.0624dW_4$$
$$dS_2 = 0.1294dt + 0.1122dW_1 + 0.1039dW_2 + 0.0616dW_3 + 0.0849dW_4$$
$$dS_3 = 0.4654dt + 0.1503dW_1 + 0.0616dW_2 + 0.0894dW_3 + 0.1323dW_4$$
$$dS_4 = 6.5328dt + 0.0624dW_1 + 0.0849dW_2 + 0.1323dW_3 + 0.37dW_4$$

Table 1. Values of initial stock prices, volatility, Drift and variance for stock (1) and stock (2)

$S_0(1)$	Volatility (σ_1)	Drift (μ_1)	Variance (Var_1)	$S_0(2)$	Volatility (σ_2)	Drift (μ_2)	Variance (Var_2)
20.71	0.3986	0.07983	60.0005	22.52	0.0000	0.00036	0.0000
21.66	0.4895	0.12017	0.9582	22.52	0.0000	0.00036	0.0000
18.90	0.9177	0.42149	3.3683	22.52	0.0000	0.00036	0.0000
21.49	0.0000	0.00038	0.0000	21.49	0.0000	0.00036	0.0000
21.49	0.2591	0.03394	0.2684	22.52	0.0000	0.00036	0.0000
20.00	0.0000	0.00041	0.0000	20.00	0.2425	0.02979	0.2352
20.00	0.0000	0.00041	0.0000	21.00	0.2425	0.02940	0.2352
20.00	0.0000	0.00041	0.0000	20.38	0.155	0.01241	0.0961

20.00	0.3569	0.06409	0.5094	20.10	0.03465	0.000998	0.0048
20.01	0.0025	0.00041	0.00003	20.02	0.0000	0.000406	0.0000
19.50	0.1184	0.00743	0.05603	21.35	0.2125	0.02296	0.1806
19.07	0.2734	0.03779	0.2989	20.50	0.0000	0.000396	0.0000
19.52	0.1301	0.00889	0.06765	20.50	0.2454	0.03050	0.2408
19.00	0.125	0.00824	0.0625	21.35	0.0000	0.000380	0.0000
19.50	0.0000	0.00042	0.0000	21.35	0.0000	0.000380	0.0000

Table 2. Values of initial stock prices, volatility, Drift and variance for stock (3) and stock (4)

$S_0(3)$	Volatility (σ_3)	Drift (μ_3)	Variance (Var_3)	$S_0(4)$	Volatility (σ_4)	Drift (μ_4)	Variance (Var_4)
21.02	0.3501	0.061689	0.4903	21.01	0.1275	0.03289	0.0650
19.32	0.52	0.1356	1.0816	20.00	0.12	0.02921	0.0576
21.40	0.3882	0.07574	0.6028	18.50	0.6336	0.8032	1.6055
19.95	0.0000	0.000407	0.0000	19.90	0.6062	0.7353	1.4699
20.57	0.0000	0.000395	0.0000	20.00	0.4295	0.3693	0.7377
21.50	0.0000	0.00038	0.0000	19.90	0.0000	0.0004	0.0000
21.00	0.0000	0.00038	0.0000	19.00	0.0000	0.0004	0.0000
20.47	0.1325	0.00092	0.07023	19.90	0.2250	0.1017	0.2025
21.50	0.2974	0.04461	0.3537	19.90	0.0000	0.0004	0.0000
21.57	0.0175	0.00053	0.00123	19.90	0.0289	0.000207	0.0003
22.60	0.275	0.03822	0.3025	15.03	0.4660	0.4348	0.8686
22.00	0.125	0.00819	0.0625	17.00	0.2246	0.1014	0.2018
21.50	0.2264	0.02601	0.2050	19.90	1.4001	3.9207	7.8406
22.15	0.0000	0.00037	0.0000	15.05	0.0000	0.0005	0.0000
21.10	0.3559	0.06370	0.5067	15.05	0.0000	0.0005	0.0000

Table 3. Comparison of stocks of different companies

Stock(1)	Stock(2)	Stock(3)	Stock(4)
60.0005	0.0000	0.4903	0.0650
0.9582	0.0000	1.0816	0.0576
3.3683	0.0000	0.6028	1.6055
0.0000	0.0000	0.0000	1.4699
0.2684	0.0000	0.0000	0.7377
0.0000	0.2352	0.0000	0.0000
0.0000	0.2352	0.0000	0.0000
0.0000	0.0961	0.07023	0.2025
0.5094	0.0048	0.3537	0.0000
0.00003	0.0000	0.00123	0.0003
0.05603	0.1806	0.3025	0.8686
0.2989	0.0000	0.0625	0.2018

0.06765	0.2408	0.2050	7.8406
0.0625	0.0000	0.0000	0.0000
0.0000	0.0000	0.5067	0.0000

Using equation (15) gives the following stock correlation matrix solution

$$R = \begin{pmatrix} 1.0000 & 0.3984 & 0.3985 & -0.7511 \\ 0.3984 & 1.0000 & 0.2564 & -0.3670 \\ 0.3985 & 0.2564 & 1.0000 & 0.2905 \\ -0.7511 & -0.3670 & 0.2905 & 1.0000 \end{pmatrix}$$

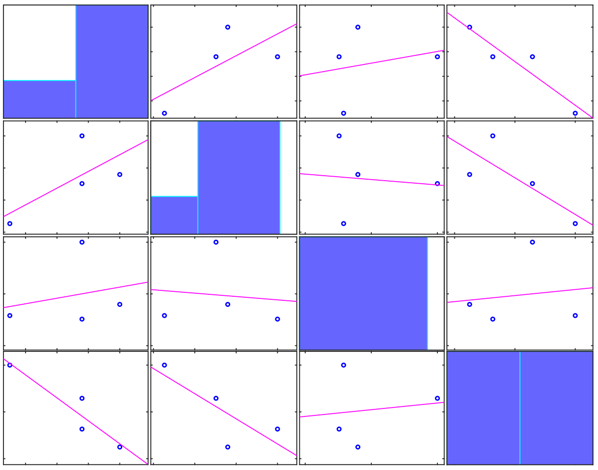

Figure 1. Graphical representation of stocks correlation matrix solution.

Adopting Section (2.1) gives the following below

$H_0 : \rho = \rho_0$

$H_0 : \rho \neq \rho_0$

Using (1.19) and (1.20) gives

$$\overline{r}_1 = \frac{1}{3}(0.3984 + 0.3985 - 0.7511) = 0.0153, \overline{r}_2 = 0.0959, \overline{r}_3 = 0.3151, \overline{r}_4 = -0.2759$$

$$\overline{r} = \frac{2}{4(3)}(0.3984 + 0.3985 - 0.7511 + 0.2564 - 0.3670 + 0.2905) = 0.0376$$

$$\sum_{1<}\sum_{k}(r_{ik} - \overline{r})^2 = (0.3984 - 0.0376)^2 + \cdots + (0.2905 - 0.0376)^2 = 1.0225$$

$$\sum_{k=1}^{4}(\overline{r}_k - \overline{r})^2 = (0.0153 - 0.0376)^2 + \cdots + (-0.2759 - 0.0376)^2 = 0.1792$$

$$\hat{r} = \frac{(4-1)^2\left(1-(1-0.0376)^2\right)}{4-(4-2)(1-0.0376)^2} = 0.3092$$

$$T = \frac{(60-1)}{(1-0.0376)^2}\left[1.0225 - (0.3092)(0.1792)\right] = 61.6039$$

Since $(\rho+1)(\rho-1)/2 = 5(2) = 5$, the 5% critical value for the test in () is $\chi^2_{5(0.05)(0.05)} = 11.07$

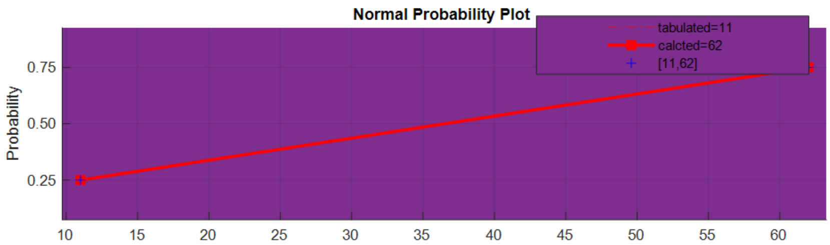

Figure 2. Graphical representation of normal distribution.

4. Discussion

To illustrate the stock market price fluctuations using our proposed model, we use daily prices in naira of four (4) selected stocks for sixty (60) trading days from Nigeria stock Exchange (NSE) extracted from [7]. The stock prices were partitioned into four (4) compartments each to having a total of fifteen (15) compartments in sixty (60) trading days, see appendix 1.

The initial stock prices were taken from each of the respective compartments to give a total of fifteen (15) observations for different stocks (i.e. $S_0(1),\cdots,S_0(4)$). The volatility, drift and variance coefficients were obtained from each of these four (4) different stocks (i.e. S_i's, $\text{i}=1,2,3,4$ using equations (15)-(19)). The trading days were taken to be $\sqrt{60}$. These computations were made using the stochastic part which is the function of stock volatility, see Table 1 and 2 respectively.

In Table 3, using our criteria in Section 2.1 shows that stock (1) gives the best investment returns. This result is in consonants with the result of [7]. It implies that investors can eventually invest on Stock (1) in order to maximize profit and minimize loss; which is the expectation of every investor.

In Figure 1 above shows the degree of relationships existing between different stocks. They are correlated with equal variances displaying good agreement in the stock expected returns of an investor.

From Goodness of fit test of the stock market prices shows that the computed value is greater than the tabulated value. We therefore reject the null hypothesis. This implies that the stock returns are statistically significant and follows normal distributions. This is highly reliable because; it describes the stock volatility as time dependent variable which is characterized by random features.

Figure 2 is just to attest for the adequacy of the propose model. The normal distribution is imperative because it makes statistics a lot easier, and more practicable

5. Conclusion

This paper investigated the problem of stock market price fluctuations on expected returns for investors. The detailed conditions for obtaining the drifts, volatilities and variances of four different stocks were considered. We compared the variances of four different stocks using our criteria for selection and the result showed that stock (1) is the best among the stocks of different companies. From the stochastic analysis of the model; systems of non-linear ordinary differential equations were developed by means of covariance matrix on the stochastic part of the expected returns of investors while the deterministic part a function of the drift parameter as a column vector. We imposed a constraint that led the drift parameter to zero and working with the stochastic part of the equation by exploring correlation matrix solution a statistical goodness of fit showed stock returns are statistically significant, correlated and follows a normal distribution. The test is shown to be adequate and reliable so that investors can comfortably invest for the purpose of profit making.

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Appendix

Table 1

S_1	20.71	21.60	20.63	19.65
S_2	22.52	22.52	22.52	22.52
S_3	21.02	20.02	20.00	19.32
S_4	21.01	21.52	21.52	21.52

Table 2

S_1	21.66	22.80	22.87	20.83
S_2	22.52	22.52	22.52	22.52
S_3	19.32	21.40	21.40	21.40
S_4	20.00	20.00	20.00	19.52

Table 3

S_1	18.70	18.00	21.49	21.49
S_2	22.52	22.52	22.52	22.52
S_3	21.40	20.39	20.47	19.50
S_4	18.50	17.27	19.90	19.90

Table 4

S_1	21.49	21.49	21.49	21.49
S_2	22.52	22.52	22.52	22.52
S_3	19.95	19.95	19.95	19.95
S_4	19.90	19.90	22.00	22.00

Table 5

S_1	21.49	21.70	20.71	20.71
S_2	22.52	22.52	22.52	22.52
S_3	20.57	20.57	20.57	20.57
S_4	20.00	19.20	19.20	21.01

Table 6

S_1	20.00	20.00	20.00	20.00
S_2	20.02	20.99	20.99	20.99
S_3	21.50	21.50	21.50	21.00
S_4	19.90	19.90	19.90	19.90

Table 7

S_1	20.00	20.00	20.00	20.00
S_2	20.02	20.99	20.99	20.99
S_3	21.50	21.50	21.50	21.00
S_4	19.90	19.90	19.90	19.90

Table 8

S_1	20.00	20.00	20.00	20.00
S_2	21.00	20.38	20.38	20.38
S_3	21.00	20.47	20.47	20.47
S_4	19.00	19.90	19.90	19.90

Table 9

S_1	20.00	20.42	19.01	19.01
S_2	20.38	20.38	20.50	20.50
S_3	20.47	20.47	21.50	21.50
S_4	19.90	19.90	19.90	19.90

Table 10

S_1	20.01	20.01	20.01	20.00
S_2	20.02	20.02	20.02	20.02
S_3	21.57	21.57	21.57	21.50
S_4	19.90	19.90	20.00	20.00

Table 11

S_1	19.50	19.01	19.01	19.07
S_2	21.35	21.35	21.35	20.50
S_3	22.60	22.60	22.60	21.50
S_4	15.03	16.50	17.00	17.00

Table 12

S_1	19.07	19.8	19.44	18.52
S_2	20.50	20.5	20.5	20.5
S_3	22.00	21.5	21.5	21.5
S_4	17.00	17	17.57	17.91

Table 13

S_1	19.52	19.01	18.99	19
S_2	20.5	20.5	21.35	21.35
S_3	21.50	21.50	21.05	22.15
S_4	19.90	19.90	15.05	15.05

Table 14

S_1	19	19	19	19.5
S_2	21.35	21.35	21.35	21.35
S_3	22.15	22.15	22.15	22.15
S_4	15.05	15.05	15.05	15.05

Table 15

S_1	19.5	19.5	19.5	19.5
S_2	21.35	21.35	21.35	21.35
S_3	21.1	21.5	22.5	22.5
S_4	15.05	15.05	15.05	15.05