The Analytical Solution Via Cole-Hopf Transformation of the Unsteady MHD Equations of Salt Water Flow Subjected to Multiple Transport Effects

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Abstract

The purpose of this study is to examine the effect of MHD properties on the unsteady hydro-magnetic natural convection with heat transfer over an impulsively moving vertical plate embedded in salt water. The whole system rotates about an axis normal to the plane of the plate with uniform angular velocity. A variable applied magnetic field and Hall current affect the incompressible and optically thick fluid presented by sea water as the working fluid in our problem. A new manipulation as a single independent variable is introduced to reduce the system of ordinary differential equations by successive eliminations to one nonlinear ODE for the secondary velocity, under simple initial and boundary conditions, which is solved analytically using the Cole-Hopf transformation. Through this study, it was found that the resulting solutions include the non-dimensional temperature, primary and secondary velocities depend on different governing parameters. The results indicate that all these quantities are damped far from the rotating plate at different values of the governing parameters. From the irreversible thermodynamic point of view, the entropy generation and Bejan numbers are estimated and illustrated under the same conditions. The results of this research may find its echo in different vital applications such as heat exchangers, Hall accelerators and power generators.

Keywords

Salt water, Unsteady MHD equations, Cole-Hopf transformation, Heat Transfer, Mechanical Rotation, Thermal radiation, Hall Effect

1. Introduction

The rotating unsteady Magnetohydrodynamic (MHD) convection heat transfer with or without Hall currents is of a technical importance especially in fluid engineering, geophysics and astrophysics. The effect of transversely applied magnetic field on convection flows of an electrically conducting fluid has been discussed by several authors notably V. M. Soundalgekar [1], and H. A. Attia and N. A. Kotb [2], G. S. Seth, et al. [3] looked at the effects of Hall current and rotation on MHD natural convection flow past an impulsively moving vertical plate with ramped temperature in the presence of thermal diffusion with heat absorption. Even though it is of considerable importance to study how the results of the hydrodynamic problems get modified by the effects of Hall currents. A comprehensive discussion of Hall currents is given by T.G. Cowling in [4]. S. M. Musundi, et al. in [5], studied magnetic field and Hall current effects on the MHD free convection flow past a vertical rotating flat plate. They considered a case where a strong magnetic field is applied at an angle to both the electric field and the direction of flow of the fluid.

Inserting the effect of thermal radiation into the governing equations makes their solution more difficult. Thus, Rosseland approximation for the radiative flux vector is applied in order to simplify the energy equation. Viskanta and Grosh are the pioneering investigators who study the effects of thermal radiation on temperature distribution and heat transfer in an absorbing and emitting media. They applied Rosseland approximation as the fluid flow over a wedge [6].

In our study, the effect of strong uniform applied magnetic and radiation fields on an electrically conducting, optically thick radiating incompressible fluid (Salt water) has been investigated. The whole system rotates mechanically about an axis normal to the plane of the plate with uniform angular velocity, Ω^* as shown in Figure 1. The plate is moved impulsively with an initial velocity in the y -direction before starting its mechanical rotation.

Most of the previous studies are lack of studying the magneto-hydrodynamic and thermodynamic properties under the above mentioned conditions for salt water as the working fluid, here we apply a pure analytical method via Cole-Hopf transformation as a relevant task. The illustrations are done relying on the available data for sea water.

The paper is organized as follows: Section 2 introduces the physical problem and basic equations, Section 3 describes the method of solution, Section 4 represents the results and discussion and Section 5 represents the conclusion.

2. The Physical Problem and Basic Equations:

In our problem, the unsteady hydromagnetic natural convection flow with heat transfer and Hall currents of an electrically conducting fluid (Salt water) is studied. The fluid is viscous, incompressible, Newtonian and optically thick radiating. The fluid flow is along a vertical $\bar{x}$ - $\bar{y}$ plane plate. The plate is of infinite extension the in $\bar{x}$ and $\bar{y}$ directions and is electrically non-conducting; all physical quantities depend on $\bar{z}$ and $\bar{t}$ only. Also no external electric field is applied; so the effect of polarization of fluid is negligible according to G. S. Seth, et al. in [7].

As shown in Figure 1 can be seen a uniform strong magnetic field. If the applied magnetic field is perpendicular to the $\bar{x} - \text{axis}$, thus $\varphi = 90^\circ$. Hence, the effective magnetic field that generates the Hall current density vector J is the horizontal component $B_{\bar{z}}$ according to Jacob et al. [8]. In this study, it is assumed that the induced magnetic field generated by fluid motion is negligible in comparison to the applied one according to Cramer and Pai [9], such that $\vec{B}_{\bar{z}} = \varepsilon B_{\bar{z}} \vec{k} = B_{\bar{z}} \sin \varphi \vec{k} = B_{\bar{z}} \vec{k}$.

Figure 1. The Flow Geometry.

At $\bar{t} = 0$, both the plate and the fluid start an impulsive motion in $\bar{y}$ -direction with an initial secondary velocity $\bar{v}_0$ in its own plane.

At $\bar{t} > 0$, the entire system rotates about the $\bar{z}$ -axis with a uniform angular velocity Ω^* according to Seth et al., in [6].

The basic equations for the flow of electrically incompressible and optically thick radiating fluid under a strong variable transverse magnetic field and rotation are given by Musundi, S. M. et al., in [7] and G. S. Seth et al., in [6].

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + 2\Omega^* \bar{v} + g\beta_1(T - T_\infty) + \frac{\sigma \varepsilon^2 B^2 (m\varepsilon \bar{v} - \bar{u})}{\rho(\varepsilon^2 m^2 + 1)} \quad (1)$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} = \nu \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} - 2\Omega^* \bar{u} - \frac{\sigma \varepsilon^2 B^2 (m\varepsilon \bar{u} + \bar{v})}{\rho(\varepsilon^2 m^2 + 1)}, \quad (2)$$

$$\frac{\partial T}{\partial \bar{t}} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial \bar{z}^2} + \frac{\mu}{c_p} \left(\left(\frac{\partial \bar{u}}{\partial \bar{z}} \right)^2 + \left(\frac{\partial \bar{v}}{\partial \bar{z}} \right)^2 \right) - \frac{\partial q_r}{\partial \bar{z}}, \quad (3)$$

where, $\bar{u}$ and $\bar{v}$ are the dimensional primary, secondary velocities respectively.

The dimensional initial and boundary conditions can be written as:

$$\text{At } \bar{z} = 0, \bar{t} = 0: T = T_0, \bar{v} = \bar{v}_0, \text{ while as } \bar{z} \rightarrow \infty, \bar{t} > 0: T = T_\infty, \bar{u} \rightarrow 0, \bar{v} \rightarrow 0.$$

For an optically thick fluid, in addition to heat radiation emission, there is also self-absorption and usually the radiation absorption coefficient is wavelength dependent and large so we can adopt the Rosseland approximation for thermal radiative flux vector q_r according to Sparrow and Cess in [9].

$$q_r = -\frac{4\sigma^*}{3\alpha_1} \frac{\partial T^4}{\partial \bar{z}}, \quad (4)$$

Here T^4 is expanded in Taylor series about free stream temperature T_∞ ,

$$T^4 = 4 T_\infty^3 \frac{\partial T}{\partial \bar{z}} - 3 T_\infty^4. \quad (5)$$

After neglecting second and higher order terms in $(T-T_\infty)$, this equation reduces to:

$$T^4 = 4 T_\infty^3 \frac{\partial T}{\partial \bar{z}}.$$

Equation (3) reduces to:

$$\frac{\partial T}{\partial \bar{t}} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial \bar{z}^2} + \frac{\mu}{C_p} \left(\left(\frac{\partial \bar{u}}{\partial \bar{z}} \right)^2 + \left(\frac{\partial \bar{v}}{\partial \bar{z}} \right)^2 \right) + \frac{16 \sigma^* T_\infty^3}{3 \rho C_p \alpha_1} \frac{\partial^2 T}{\partial \bar{z}^2}. \quad (6)$$

The mean absorption coefficient (α_1) for seawater according to Hilliard et al. [10] is defined as:

$$\alpha_1 = 0.0037 d \sqrt{f}, \quad (7)$$

where f is the frequency of the electromagnetic radiation in cycles per second, d is the depth of penetration in meters.

The frequency at which the total attenuation is a minimum for any fixed depth (d) is given by [10]:

$$f = \left(\frac{478}{d} \right)^{1.76}. \quad (8)$$

The flow governing equations (1), (2) and (6) are transformed into their non-dimensional form by applying the following similarity variables:

$$z = \frac{\bar{z}}{L}, t = \frac{\bar{t}}{t_0} = \frac{\bar{t}}{L/U}, u = \frac{\bar{u}}{U}, v = \frac{\bar{v}}{U}, \Omega = \frac{\Omega^* L}{U},$$

$$\theta = \frac{T-T_\infty}{T_w-T_\infty} = \frac{T-T_\infty}{\Delta T} \Rightarrow T = \Delta T \theta + T_\infty. \quad (9)$$

Thus, after introducing these expressions and making some algebraic manipulations we get the following non-dimensional partial differential equations in three dependent variables namely $u = u(z, t)$, $v = v(z, t)$ and $\theta = \theta(z, t)$ respectively:

$$\frac{\partial u}{\partial t} = \frac{1}{Re} \frac{\partial^2 u}{\partial z^2} + 2\Omega v + Gr\theta + \frac{\beta \varepsilon^2 (m\varepsilon v - u)}{(\varepsilon^2 m^2 + 1)}, \quad (10)$$

$$\frac{\partial v}{\partial t} = \frac{1}{Re} \frac{\partial^2 v}{\partial z^2} - 2\Omega u - \frac{\beta \varepsilon^2 (m\varepsilon u + v)}{(\varepsilon^2 m^2 + 1)}, \quad (11)$$

$$\frac{\partial \theta}{\partial t} = \frac{(1+n)}{Pr} \frac{\partial^2 \theta}{\partial z^2} + Ec \left(\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right). \quad (12)$$

Appendix (A) contains the definitions of the transport coefficients and parameters mentioned in equations (10-12), see also [6, 7].

Transforming the non-dimensional partial differential equations (10), (11) and (12) with respect to a new non-dimensional independent variable ξ by introducing the following ansatz:

$$\xi = \xi(z, t) = z - wt, \quad (13)$$

where, ξ is non-dimensionalized such that $\xi = \frac{\bar{\xi}}{L}$; the non-dimensional velocity $w = \frac{\bar{w}}{U}$. Thus,

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial \xi} \frac{\partial \xi}{\partial t} = -w \frac{du(\xi)}{d\xi}, \frac{\partial v}{\partial t} = \frac{\partial v}{\partial \xi} \frac{\partial \xi}{\partial t} = -w \frac{dv(\xi)}{d\xi}, \frac{\partial \theta}{\partial t} = \frac{\partial \theta}{\partial \xi} \frac{\partial \xi}{\partial t} = -w \frac{d\theta(\xi)}{d\xi}, \quad (14)$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial \xi} \frac{\partial \xi}{\partial z} = \frac{du(\xi)}{d\xi}, \frac{\partial v}{\partial z} = \frac{\partial v}{\partial \xi} \frac{\partial \xi}{\partial z} = \frac{dv(\xi)}{d\xi}, \quad \frac{\partial \theta}{\partial z} = \frac{\partial \theta}{\partial \xi} \frac{\partial \xi}{\partial z} = \frac{d\theta(\xi)}{d\xi},$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{d^2 u(\xi)}{d\xi^2}, \frac{\partial^2 v}{\partial z^2} = \frac{d^2 v(\xi)}{d\xi^2}, \frac{\partial^2 \theta}{\partial z^2} = \frac{d^2 \theta(\xi)}{d\xi^2}.$$

Hence obtaining the flow governing equations in the new independent variable ξ , after substituting expressions (14) into equations (10-12) gives

$$w \frac{du(\xi)}{d\xi} + \frac{1}{\lambda} \frac{d^2 u(\xi)}{d\xi^2} + av(\xi) + b\theta(\xi) + cv(\xi) - qu(\xi) = 0, \quad (15)$$

$$w \frac{dv(\xi)}{d\xi} + \frac{1}{\lambda} \frac{d^2v(\xi)}{d\xi^2} - au(\xi) - cu(\xi) - qv(\xi) = 0 \quad (16)$$

$$w \frac{dv(\xi)}{d\xi} + \frac{1}{\lambda} \frac{d^2v(\xi)}{d\xi^2} - au(\xi) - cu(\xi) - qv(\xi) = 0 \quad (17)$$

where, for brevity writing:

$$\lambda = Re, a = 2\Omega, b = Gr, c = \frac{\beta \varepsilon^3 m}{(\varepsilon^2 m^2 + 1)}, q = \frac{\beta \varepsilon^2}{(\varepsilon^2 m^2 + 1)}, \epsilon = Pr^{-1}, l = Ec, = Pr W = \frac{16 \sigma^* T_\infty^3}{3 \alpha_1 k}.$$

Our task is to reduce the system of ordinary differential equations (15-17) by successive eliminations in terms of the dependent variable $v(\xi)$ under simple initial and boundary conditions. This will lead to:

$$u(\xi) = \frac{1}{(a+c)\lambda} \left(-q\lambda v(\xi) + w\lambda \frac{dv(\xi)}{d\xi} + \frac{d^2v(\xi)}{d\xi^2} \right), \quad (18)$$

$$\theta(\xi) = \frac{a_8}{w} v(\xi) + \frac{a_9}{w} \frac{dv(\xi)}{d\xi} + a_{13} \frac{d^2v(\xi)}{d\xi^2} - a_{14} \frac{d^3v(\xi)}{d\xi^3} - a_{15} \frac{d^4v(\xi)}{d\xi^4} \quad (19)$$

Therefore we obtain the following nonlinear ODE in the dependent variable $v(\xi)$;

$$a_1 \left(\frac{dv(\xi)}{d\xi} \right)^2 + a_2 \left(\frac{d^2v(\xi)}{d\xi^2} \right)^2 + a_3 \frac{d^3v(\xi)}{d\xi^3} + a_4 \left(\frac{d^3v(\xi)}{d\xi^3} \right)^2 + \frac{dv(\xi)}{d\xi} \left(a_8 - a_{11} \frac{d^2v(\xi)}{d\xi^2} - a_{12} \frac{d^3v(\xi)}{d\xi^3} \right) + \frac{d^2v(\xi)}{d\xi^2} \left(a_9 + \frac{a_8}{w} a_{16} + a_{10} \frac{d^3v(\xi)}{d\xi^3} \right) + a_5 \frac{d^4v(\xi)}{d\xi^4} + a_6 \frac{d^5v(\xi)}{d\xi^5} - a_7 \frac{d^6v(\xi)}{d\xi^6} = 0, \quad (20)$$

where,

$$\begin{aligned} a_1 &= \left(l + \frac{lq^2}{(a+c)^2} \right), a_2 = \frac{lw^2}{(a+c)^2}, a_3 = \left(\frac{2qw a_{16}}{b(a+c)} + wa_{13} \right), \\ a_4 &= \frac{l}{(a+c)^2 \lambda^2}, a_5 = \left(-\frac{2w^2}{b(a+c)\lambda} + \frac{a_{16}(2q - w^2\lambda)}{b(a+c)\lambda} \right), \\ a_6 &= \left(-\frac{w}{b(a+c)\lambda^2} - \frac{2w a_{16}}{b(a+c)\lambda} \right), a_7 = \frac{a_{16}}{b(a+c)\lambda^2}, \\ a_8 &= -\frac{(a^2 + 2ac + c^2 + q^2)w}{b(a+c)}, a_9 = \frac{2qw^2}{b(a+c)}, a_{10} = \frac{2lw}{(a+c)^2 \lambda}, \\ a_{11} &= \frac{2lqw}{(a+c)^2}, a_{12} = \frac{2lq}{(a+c)^2 \lambda}, a_{13} = \frac{(2q - w^2\lambda)}{b(a+c)\lambda}, \\ a_{14} &= \frac{2w}{b(a+c)\lambda}, a_{15} = \frac{1}{b(a+c)\lambda^2}, a_{16} = (\epsilon + n\epsilon) \end{aligned}$$

With the initial and boundary conditions:

$$\xi(z=0, t=0) = 0 \Rightarrow v(0) = v_0 = \bar{v}_0/U; \text{ while as } \xi(z \rightarrow \infty, t > 0) \rightarrow \infty \Rightarrow \theta \rightarrow 0, u \rightarrow 0, v \rightarrow 0.$$

3. Method of solution

We suggest a method to solve equation (20) by introducing the *Cole-Hopf transformation* [11-13], which will give an analytical solution of this equation.

1) Let us advent a finite polynomial of *Cole-Hopf transformations* of the velocity component $v(\xi)$, its finite series is in the form:

$$v(\xi) = \sum_{j=0}^{n<\infty} A_j \frac{\partial^j}{\partial \xi^j} (\log(\sigma(\xi))) , \quad (21)$$

where the coefficients A_j are real constants. The core function $\sigma(\xi)$ is assumed continuous, with damping values as ξ increases, it vanishes as $\xi \rightarrow \infty$. Each term in the summation represents an individual Cole-Hopf transformation of increasing order.

After some trials, we found out that the second order term as a suitable resort to the solution,

$$v(\xi) = A \frac{\partial^2}{\partial \xi^2} (\log(\sigma(\xi))), \quad (22)$$

which satisfies the basic needs of the suitable physical situation of the problem as is will be seen later in details.

Substituting eq. (22) into eq. (20), a number of independent algebraic equations each of which corresponds to a specific power of the inverse of $(\sigma(\xi))$ is obtained.

This is done by the help of symbolic software, see details in [13]. The equation corresponding to $(\sigma(\xi))^{-9}$ is

$$w\lambda \frac{d\sigma(\xi)}{d\xi} + 10 \frac{d^2\sigma(\xi)}{d\xi^2} = 0. \quad (23)$$

Its solution for the core function $\sigma(\xi)$ is obtained

$$\sigma(\xi) = -\frac{10 e^{-\frac{1}{10}w\lambda\xi} c_1}{w\lambda} + C_2 \quad (24)$$

Substituting the core function $\sigma(\xi)$ into (22) to obtain the non-dimensional secondary velocity:

$$v(\xi) = A \left(-\frac{c_1 e^{-\frac{1}{10}w\lambda\xi} w^3 \lambda^3}{10(-10 c_1 + e^{-\frac{1}{10}w\lambda\xi} w\lambda)^2} \right) : c_1 = \frac{C_1}{C_2}. \quad (25)$$

Substituting from equation (25) into equations (22) and (21) for the primary velocity and temperature respectively:

$$u(\xi) = (c_1 e^{-\frac{1}{10}w\lambda\xi} w^3 \lambda^3 (100c_1^2 (100q - 11w^2\lambda) - 40c_1 e^{-\frac{1}{10}w\lambda\xi} w\lambda (50q + w^2\lambda) + e^{-\frac{1}{10}w\lambda\xi} w^2 \lambda^2 (100q + 9w^2\lambda))) / (1000(a + c)(-10c_1 + e^{-\frac{1}{10}w\lambda\xi} w\lambda)^4) \quad (26)$$

$$\theta(\xi) = (c_1 e^{-\frac{1}{10}w\lambda\xi} w^3 \lambda^3 (e^{-\frac{1}{10}w\lambda\xi} A_0 A_1 + 600c_1^2 e^{-\frac{1}{5}w\lambda\xi} w^2 \lambda^2 A_2 - 20c_1 e^{-\frac{3}{10}w\lambda\xi} w^3 \lambda^3 A_3 + 10000c_1^4 A_4 + e^{-\frac{2}{5}w\lambda\xi} w^4 \lambda^4 A_5)) \div (A_6 (-10c_1 + e^{-\frac{1}{10}w\lambda\xi} w\lambda)^6). \quad (27)$$

where,

$$A_0 = -2000c_1^3 w\lambda, A_1 = (20000((a + c)^2 + q^2) - 1800qw^2\lambda - 213w^4\lambda^2)$$

$$A_2 = (10000((a + c)^2 + q^2) + 200qw^2\lambda - 89w^4\lambda^2),$$

$$A_3 = (20000((a + c)^2 + q^2) + 2200qw^2\lambda - 13w^4\lambda^2),$$

$$A_4 = (10000(a + c)^2 + (100q - 11w^2\lambda)^2),$$

$$A_5 = (10000(a + c)^2 + (100q + 9w^2\lambda)^2), \text{ and}$$

$$A_6 = 100000b(a + c).$$

2) The determinations of A, w and c_1

The core function $\sigma(\xi)$ in (24) satisfies any one of the independent equations corresponding to the different powers of $(\sigma(\xi))^{-1}$: one of these equations corresponds to the power of $(\sigma(\xi))^{-8}$:

$$\left((420a(1 + n)\epsilon + 420c(1 + n)\epsilon + b\lambda A(-8q + 3w^2\lambda)) \left(\frac{d\sigma(\xi)}{d\xi} \right)^2 + 420b\lambda \left(\frac{d^2\sigma(\xi)}{d\xi^2} \right)^2 + 4b\lambda A \frac{d\sigma(\xi)}{d\xi} \left(27w\lambda \frac{d^2\sigma(\xi)}{d\xi^2} + 20 \frac{d^3\sigma(\xi)}{d\xi^3} \right) \right) = 0,$$

the other one corresponds to the power of $(\sigma(\xi))^{-7}$:

$$\begin{aligned} & w(-b\lambda q A \lambda^2 + 30a(1 + 2(1 + n)\epsilon\lambda) + 30c(1 + 2(1 + n)\epsilon\lambda)) \left(\frac{d\sigma(\xi)}{d\xi} \right)^3 + 150b\lambda A \left(\frac{d^2\sigma(\xi)}{d\xi^2} \right)^3 \\ & + 3b\lambda A \frac{d\sigma(\xi)}{d\xi} \frac{d^2\sigma(\xi)}{d\xi^2} \left(27w\lambda \frac{d^2\sigma(\xi)}{d\xi^2} + 40 \frac{d^3\sigma(\xi)}{d\xi^3} \right) \\ & + 2 \left(\frac{d\sigma(\xi)}{d\xi} \right)^2 \left((420a(1 + n)\epsilon + 420c(1 + n)\epsilon + b\lambda A(-8q + 3w^2\lambda)) \frac{d^2\sigma(\xi)}{d\xi^2} \right. \\ & \left. + b\lambda A \left(9w\lambda \frac{d^3\sigma(\xi)}{d\xi^3} + 5 \frac{d^4\sigma(\xi)}{d\xi^4} \right) \right) = 0. \end{aligned}$$

Therefore, two algebraic equations are obtained namely

$$(1050a(1+n)\epsilon + 1050c(1+n)\epsilon - blA\lambda(20q + 7w^2\lambda)) = 0 \quad (28)$$

$$blA\lambda^2(60q + 11w^2\lambda) - 600a(-5 + 4(1+n)\epsilon\lambda) - 600c(-5 + 4(1+n)\epsilon\lambda) = 0 \quad (29)$$

Their simultaneous solutions give for both w and A the following values:

$$w = -\frac{2\sqrt{\frac{5}{7}\sqrt{4q+q\epsilon\lambda+nq\epsilon\lambda}}}{\sqrt{-4\lambda+\epsilon\lambda^2+n\epsilon\lambda^2}}, A = \frac{105(a+c)(-4+\epsilon\lambda+n\epsilon\lambda)}{4blq\lambda^2} \quad (30)$$

$$w = \frac{2\sqrt{\frac{5}{7}\sqrt{4q+q\epsilon\lambda+nq\epsilon\lambda}}}{\sqrt{-4\lambda+\epsilon\lambda^2+n\epsilon\lambda^2}}, A = \frac{105(a+c)(-4+\epsilon\lambda+n\epsilon\lambda)}{4blq\lambda^2} \quad (31)$$

Using the initial condition $v(\xi(z, t = 0) = 0) = v_0$, to obtain the following equation:

$$10v_0(-10c1 + w\lambda)^2 + c1w^3A\lambda^3 = 0. \quad (32)$$

By inspection, substituting from eq. (31) into eq. (32), obtaining:

$$\frac{10}{7} \left(\frac{15\sqrt{35}(a+c)c1(q(4+(1+n)\epsilon\lambda))^{3/2}}{blq\sqrt{\lambda(-4+(1+n)\epsilon\lambda)}} + 7(10c1 - \frac{2\sqrt{\frac{5}{7}\sqrt{q(4+(1+n)\epsilon\lambda)}}}{\sqrt{\lambda(-4+(1+n)\epsilon\lambda)}})^2 v_0 \right) = 0 \quad (33)$$

Solving eq. (33), to economize some space, the physically valid root of $c1$ is retained together with (31):

$$c1 = \frac{1}{8\sqrt{35}bl\lambda(-4+(1+n)\epsilon\lambda)v_0} (-12c\sqrt{\lambda(-4+(1+n)\epsilon\lambda)}\sqrt{q(4+(1+n)\epsilon\lambda)} - 3c\epsilon\lambda\sqrt{\lambda(-4+(1+n)\epsilon\lambda)}\sqrt{q(4+(1+n)\epsilon\lambda)} - 3a\sqrt{\lambda(-4+(1+n)\epsilon\lambda)}(q(4+(1+n)\epsilon\lambda))^{3/2} + 8bl\lambda\sqrt{\lambda(-4+(1+n)\epsilon\lambda)}\sqrt{q(4+(1+n)\epsilon\lambda)}v_0 + \sqrt{3} \sqrt{(a+c)q\lambda(-4+(1+n)\epsilon\lambda)(4+(1+n)\epsilon\lambda)^2(3(a+c)(4+(1+n)\epsilon\lambda)) - 16bl\lambda v_0})$$

4. The Entropy Generation

We are concerned with the heat transfer in the presence of strong uniform applied magnetic field and rotation, thus the dimensional form of the volumetric entropy generation E_g is given by [14]:

$$E_g = \frac{k}{T_0^2} \left(\frac{\partial T}{\partial z} \right)^2 + \frac{\mu}{T_0} \left(\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right) + \frac{\sigma B^2 \epsilon^2}{T_0} \left(\frac{(m\epsilon \bar{v} - \bar{u})^2 + (m\epsilon \bar{u} + \bar{v})^2}{(\epsilon^2 m^2 + 1)^2} \right), \quad (34)$$

E_g is in units of J/K.m³.sec. Applying the new dimensionless variables given by eq. (9) to eq. (34), gives:

$$E_g = \frac{k}{T_0^2} \left(\frac{\Delta T}{L} \right)^2 \left(\frac{\partial \theta}{\partial z} \right)^2 + \frac{\mu}{T_0} \left(\frac{U}{L} \right)^2 \left(\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right) + \frac{\sigma B^2 \epsilon^2 U^2}{T_0} \left(\frac{(m\epsilon v - u)^2 + (m\epsilon u + v)^2}{(\epsilon^2 m^2 + 1)^2} \right). \quad (35)$$

Multiplying both sides of eq. (35) by $L^4/k_B U$, obtaining the dimensionless volumetric Entropy generation $E_{g,nond}$:

$$E_{g,nond} = \frac{k}{T_0^2} \frac{\Delta T L^2}{k_B U} \left(\frac{\partial \theta}{\partial z} \right)^2 + \frac{\mu}{T_0} \frac{U L^2}{k_B} \left(\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right) + \frac{\sigma B^2 \epsilon^2 U L^4}{k_B T_0} \left(\frac{(m\epsilon v - u)^2 + (m\epsilon u + v)^2}{(\epsilon^2 m^2 + 1)^2} \right) \quad (36)$$

where, $E_{g,nond} = \frac{L^4}{k_B U} E_g$.

Transforming the non-dimensional volumetric entropy generation (eq.36) with respect to the independent variable ξ by substituting eq.(14) in it, obtaining:

$$E_{g,nond} = S_T + S_V + S_M \quad (37)$$

where,

$$S_T = \frac{k}{T_0^2} \frac{\Delta T L^2}{k_B U} \left(\frac{d\theta(\xi)}{d\xi} \right)^2 \quad (38)$$

$$S_V = \frac{\mu}{T_0} \frac{U L^2}{k_B} \left(\left(\frac{du(\xi)}{d\xi} \right)^2 + \left(\frac{dv(\xi)}{d\xi} \right)^2 \right) \quad (39)$$

$$S_M = \frac{\sigma B^2 \varepsilon^2 U L^4}{k_B T_0} \left(\frac{(m \varepsilon v(\xi) - u(\xi))^2 + (m \varepsilon u(\xi) + v(\xi))^2}{(\varepsilon^2 m^2 + 1)^2} \right) \quad (40)$$

The BEJAN Numbers

Introducing Bejan number (Be); as a substitutional parameter for irreversibility distributions. It is defined in [15]:

$$Be_T = \frac{S_T}{S_T + S_V + S_M}; \text{ for heat transfer,} \quad (41)$$

$$Be_V = \frac{S_V}{S_T + S_V + S_M}; \text{ for viscous dissipation,} \quad (42)$$

$$Be_M = \frac{S_M}{S_T + S_V + S_M}; \text{ for magnetic dissipation.} \quad (43)$$

5. Results and Discussion

In order to analyse the effects of Hall current, rotation, thermal radiation, impulsive motion of the plate with time on the flow-field, the non-dimensional temperature, primary and secondary fluid velocities in the boundary layer region, all are computed from the analytical solutions given by the eqs. (25), (26) and (27).

They, together with the entropy generation are displayed graphically versus boundary layer coordinate z and time t in the figures (2-17) for different values of the magnetic parameter (β), rotation parameter Ω , initial secondary velocity $\bar{v}_0$ and characteristic velocity (U) = (0.3 - 0.9) meter/sec at a characteristic length (L) = characteristic depth (d) = 2 meters, initial fluid temperature $T_0 = 293\text{K}$, dimensional free stream temperature (T_∞) = 293.1K, $\Delta T = 0.001\text{K}$, mean absorption coefficient (α_1) = 0.9, thermal radiation parameter (n) = 13.9 and the dimensional time ($\bar{t} = t t_0, t_0 = \frac{L}{U}$) interval = (0 - 133) seconds. Throughout the data in several references, the physical properties of sea water at salinity of 35 g/kg and temperature 293 K are cited in Table 1 [16, 17]. Notice that the figures are illustrated according to the best view angles.

We can notice from figures (2) to (17) that the non-dimensional temperature, velocity components and entropy generation have a periodic behaviour. Also, the initial condition $\xi(z = 0, t = 0) = 0 \Rightarrow v(0) = v_0$ and the boundary condition $\xi(z \rightarrow \infty, t > 0) \rightarrow \infty \Rightarrow \theta \rightarrow 0, u \rightarrow 0, v \rightarrow 0$ are perfectly attained.

Studying the effect of the dimensional initial secondary velocity $\bar{v}_0$ on the non-dimensional velocity components, temperature and Entropy generation:

As shown in figure (2), the initial condition is attained for different values of the non-dimensional initial secondary velocity $\bar{v}_0$. As $\bar{v}_0$ increases, the dimensionless secondary velocity $v(z, t)$ is damped at a shorter distance from the plate. While the non-dimensional secondary velocity increases in the positive direction, the dimensionless primary velocity increases in the negative direction as shown in Figures 2 and 3. This behaviour is repeated alternately at certain intervals of z and t and (The values of the governing parameters in Figures 2-5 are $U = 0.5 \frac{m}{sec}, \Omega^* = 0.5 \frac{rad}{sec}, B = 0.5 \text{ Tesla}, \beta = 0.004, Ec = 0.06, Gr = 1.472 \times 10^7, Re = 9.5 \times 10^5, m = 10^{-4}, \Omega = 2$).

Table 1. The physical properties of sea water at salinity of 35 g/kg (at 1 atmosphere).

Physical quantity	Seawater at salinity of 35 g/kg
T	293 K
C_p	3,993 J kg ⁻¹ K ⁻¹
ρ	1,024.763 kg m ⁻³
μ	1.08 × 10 ⁻³ Pa s
ν	1.05 × 10 ⁻⁶ m ² s ⁻¹
μ_0	4π × 10 ⁻⁷ N A ⁻²
σ	4.788 S m ⁻¹

α	$1.46 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$
Pr	7.2
β_1	$2.07 \times 10^{-4} \text{ K}^{-1}$
k	0.596 W/m K
m	$\cong 10^{-4}$

Figure 2. Non-dimensional secondary velocity for different values of initial secondary velocity.

Figure 3. Non-dimensional primary velocity for different values of initial secondary velocity.

For low values of $\bar{v}_0$ and $t \geq 0$, the non-dimensional temperature is almost damped near the plate. At high value of $\bar{v}_0$, $\theta(z,t)$ begins to increase in the positive and negative scales as shown in the green coloured plot of Figure 4. This means that the impulsive motion of the plate becomes an effective tool in the heat transfer throughout the boundary layer.

Figure 4. Non-dimensional temperature for different values of initial secondary velocity.

Figures 5. Non-dimensional Entropy generation for different values of initial secondary velocity.

As $\bar{v}_0$ increases, a sharp increase of the non-dimensional entropy generation $E_{g,nond}$ occurs especially at $\bar{v}_0 = 0.05 \text{ m/s}$ as shown in Figure 5. This increase is clearly observed near the plate and quickly damped at a short distance

from it. At $t > 0$, it is observed that the peak progresses away from the plate.

Studying the effect of the dimensional characteristic velocity U on the non-dimensional velocity components, temperature and Entropy generation:

Increasing the dimensional characteristic velocity U leads to an increase of Reynolds number Re which means that the inertial force becomes more dominant than the viscous force acting upon the fluid throughout the boundary layer. In other words as U increases, the momentum of the fluid becomes more dominant than the viscous force which counteract the fluid motion. Increasing the fluid momentum results in increasing the turbulence and consuming the kinetic energy within the boundary layer. Accordingly, the velocity components are damped at a shorter distance from the plate within shorter characteristic time intervals t_0 as shown in Figure 6 and Figure 7. (The values of the governing parameters in Figures 6-9 are $\bar{v}_0 = 0.03 \frac{m}{sec}$, $\Omega^* = 0.5 \frac{rad}{sec}$, $B = 0.5 Tesla$, $Gr = 1.472 \times 10^7$, $m = 10^{-4}$).

Figures 6. Non-dimensional secondary velocity for different values of characteristic velocity U .

Figure 7. Non-dimensional primary velocity for different values of characteristic velocity U .

On the other hand, increasing U leads to an increase of another important parameter which is Eckert number Ec . Increasing Ec number means that the advective transport is more dominant than the heat dissipation potential within the boundary layer. Thus, for reduced heat dissipation potential and at a fixed value of $\Delta T = 0.001 K$, an increase of the non-dimensional temperature $\theta(z, t)$ is expected as shown in Figure 8.

Increasing U as mentioned previously consumes the kinetic energy of the fluid molecules resulting in a depression of the internal energy of the fluid which in turn damping $\theta(z, t)$ at a distance not far from the plate as shown in Figure 8.

Figure 8. Non-dimensional temperature for Different values characteristic velocity U .

Figure 9. Non-dimensional Entropy generation for different values of characteristic velocity U .

As discussed in the above section, increasing the dimensional characteristic velocity results in a damping of the irre-

versible processes like the friction and the heat transfer within short time intervals and at short distance from the plate. Consequently, a damping of the non-dimensional entropy generation $E_{g,nond}$ occurs for increasing U as shown in Figure 9.

Studying the effect of the dimensional uniform angular velocity Ω^* on the non-dimensional velocity components, temperature and Entropy generation:

Although rotation is known to induce secondary fluid velocity in the flow-field by suppressing the primary fluid velocity $u(z, t)$, its accelerating effect is prevalent only in the region near to the plate whereas it has a reverse effect on secondary fluid velocity $v(z, t)$ in the region away from the plate as shown in Figure 10. This is due to Coriolis force which is dominant in the region near to the axis of rotation. (The values of the governing parameters in figures (10-13): $\bar{v}_0 = 0.03 \frac{m}{sec}$, $B = 0.5 Tesla$, $\beta = 0.004$, $Ec = 0.06$, $Gr = 1.472 \times 10^7$, $Re = 9.5 \times 10^5$).

Increasing the rotation parameter is also known to induce the heat transfer throughout the boundary layer. This can be shown in Figure 12, where $\theta(z, t)$ is almost has a negative value for different values of Ω^* .

Figure 10. Non-dimensional secondary velocity for different values of uniform angular velocity.

Figure 11. Non-dimensional primary velocity for different values of uniform angular velocity.

It is also noted that increasing the rotational parameter has the effect of broadening the non-dimensional velocity components, temperature and entropy generation profiles. This means that the damping of $v(z, t)$, $u(z, t)$, $\theta(z, t)$ and $E_{g,nond}$ occurs at a larger distance away from the plate for increasing Ω^* as shown in Figures 10, 11, 12 and 13.

Figure 12. Non-dimensional temperature for different values of uniform angular.

Figure 13. Non-dimensional Entropy generation for different values of uniform angular velocity.

Studying the effect of the applied magnetic field intensity B on the non-dimensional velocity components, temperature and Entropy generation:

Increasing the applied magnetic field intensity (B) means that Hartmann number also increases and accordingly the electromagnetic force becomes more dominant than viscous force throughout the boundary layer. The applied magnetic field induces an electromotive force which in turn induces a Hall current density $\vec{j}$. The induced hallcurrent density is in the x - y plane. Thus, increasing B has the effect of the fluid advective motion along the z -axis with the progress of time as shown in Figure 14 and Figure 15. (The values of the governing parameters in figures (14-17): $\bar{v}_0 = 0.03 \frac{m}{sec}$, $\Omega^* =$

$$0.5 \frac{\text{rad}}{\text{sec}}, \text{Ec} = 0.06, \text{Gr} = 1.472 \times 10^7, \text{Re} = 9.5 \times 10^5, m = 10^{-4}, U = 0.5 \frac{\text{m}}{\text{sec}}$$

Figure 14. Non-dimensional secondary velocity for different values of applied magnetic B.

Figure 15. Non-dimensional primary velocity for different values of applied magnetic B.

According to the experimental studies cited in [18] that increasing the applied magnetic field density depresses the heat transfer throughout the boundary layer. The reduced heat transfer has the effect of increasing the difference between the fluid temperature T and the free stream temperature T_∞ and in turns increases the non-dimensional temperature θ as shown in Figure 16 for different values of B. It is also noted that the increase of B, has the effect of sharpen the non-dimensional entropygeneration and increases its value as shown in Figure 17 for different values of B.

Figure 16. Non-dimensional temperature for different values of applied magnetic field.

Figure 17. Non-dimensional Entropy generation for different values of of applied magnetic field B.

6. Bejan numbers representation

The Bejan number due to the viscous dissipation Be_V has the maximum value followed by Bejan number due to the electromagnetic force Be_M and Bejan number due to heat transfer Be_T with the minimum value. This means that Be_V has the most responsibility for irreversible processes and in turn entropy generation within the system among the other two Bejan numbers at different values of the governing parameters as shown in Figures 18-23 (The values of the governing parameters in Figures 18-20: $\bar{v}_0 = 0.03 \frac{\text{m}}{\text{sec}}, \Omega^* = 0.5 \frac{\text{rad}}{\text{sec}}, \text{Ec} = 0.06, \text{Gr} = 1.472 \times 10^7, \text{Re} = 9.5 \times 10^5, m = 10^{-4}, U = 0.5 \frac{\text{m}}{\text{sec}}$).

As a remarkable result at $\Omega^* = 5, \Omega^* = 10$, Be_V and Be_M reaches the same maximum values alternately at certain intervals of z and t . This means that both of the viscous dissipation and the electromagnetic force alternately have an equal effect on the entropy generation as shown in Figures 19, 20, 22, and 23.

Figure 18. Be_T as a function of z and t .

Figure 19. Be_V as a function of z and t .

Figure 20. Be_M as a function of z and t . Figure 21. Be_T as a function of z and t at $\Omega = 40$. Figure 22. Be_V as a function of z and t at $\Omega = 40$. Figure 23. Be_M as a function of z and t at $\Omega = 40$.

7. Conclusion

This study considers a free convective flow of a viscous, incompressible, Newtonian and optically thick radiating fluid (sea water) under the effect of a variable strong applied magnetic field, and impulsive motion of a rotating non-conductive flat plate. By a successive elimination of the dependent variables among the MHD governing equations we employed the Cole-Hopf transformation to solve the secondary velocity nonlinear ODE by the aid of symbolic software. Some main points are concluded:

- The effect of the governing parameters on the non-dimensional temperature, secondary velocity, primary velocity, entropy generation and Bejan numbers has been studied. The initial and boundary conditions have been fulfilled.
- The increase in $\bar{v}_0$ and U tighten the non-dimensional temperature, secondary velocity, primary velocity and entropy generation.
- The increase in Ω^* widens the non-dimensional temperature, secondary velocity, primary velocity and entropy generation.
- Bejan numbers have been evaluated, showing that the viscous dissipation plays the dominant role among the other two types:

$$Be_V > Be_M > Be_T$$

For future studies, we strongly recommend to tackle the same problem under other operating conditions. For example, studying the magneto-hydrodynamic, mechanical and thermodynamic properties for the fluid between two rotating plates which can be also of different geometrical configurations.

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Appendix (A)

Definitions of the non-dimensional transformation governing parameters

The parameter	Its definition	Significance
Reynolds number	$Re = \frac{UL}{\nu}$	The ratio of The inertial force to The viscous force.
Rotational parameter	$\Omega = \frac{1}{Ro} = \frac{\Omega^* L}{U}$	The ratio of the Coriolis force to the inertial force
Grashof number	$Gr = \frac{g \beta_1 \Delta T L^3}{\nu^2} \equiv \frac{g \beta_1 \Delta T L}{U^2}$	The ratio of the Buoyancy force to the viscous force
Eckert number	$Ec = \frac{U^2}{C_p \Delta T} \equiv \frac{\nu U}{C_p \Delta T L}$	The ratio of the Advective force to heat dissipation potential
Hartmann number	$Ha = BL \sqrt{\frac{\sigma}{\mu}}$	The ratio of the Electromagnetic force to the viscous force
The magnetic parameter	$\beta = \frac{B \sqrt{\mu \sigma}}{\rho U} Ha = \frac{\sigma L B^2}{\rho U}$	
Non-dimensional number	$W = \frac{16 \sigma^* T_\infty^3}{3 \rho L U C_p \alpha_1}$	
Thermal radiation parameter	$n = PrW = \frac{16 \sigma^* T_\infty^3}{3 \alpha_1 k}$	
Hall current parameter	$m = \omega_e t_e$	

Nomenclature

C_p	Isobaric specific heat, J. kg ⁻¹ . K ⁻¹
g	Acceleration due to gravity, m/s ²
S_T	The non-dimensional entropy generation corresponding to heat transfer
S_V	The non-dimensional entropy generation corresponding to viscous dissipation
S_M	The non-dimensional entropy generation corresponding to magnetic dissipation
t_0	Characteristic time, s
t_e	Electron collision time, s
k	Thermal conductivity of the fluid, W.m ⁻¹ .K ⁻¹
K_B	Boltzmann constant, J. K ⁻¹
L	Characteristic length of the system, m
m	Hall current parameter
U	Characteristic velocity of the fluid, m.s ⁻¹
v_0	Non-dimensional initial secondary velocity

Greek symbols

α	thermal diffusivity, m ² . s ⁻¹
β_1	thermal expansion coefficient, K ⁻¹
μ	Dynamic viscosity, kg. m ⁻¹ .s ⁻¹

μ_0	Magnetic permeability of free space, N A ⁻²
ν	Kinematic viscosity, m ² /s
ρ	Fluid density, kg.m ⁻³
σ	Electrical conductivity of the fluid, S/m
σ^*	Stefan–Boltzmann constant, W.m ⁻² .K ⁴
ω_e	Cyclotron frequency, s ⁻¹
Ω	Rotational parameter