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On the Misconceptions of 10th Grade Students about Analytical Geometry

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Abstract

Mathematics is an indispensable tool of science and technology and a part of everyday life. Mathematics education is the most important factor in the rational approach of individuals to analytical thinking and problem solving of log problems. Errors and misconception are one of the factors that make mathematics education difficult. The purpose of this research is to determine the relationship between the misconceptions of errors and concepts about analytical geometry, the attitudes of students towards analytic geometry, and the misconceptions of analytical geometry in order to overcome the object misconceptions. In the first stage, 2552 students in the 10th grade were studied. In the second stage, 299 students were studied and 10 students were interviewed to elicit the conceptual misconceptions from these students. The reason that the mathematical subjects are connected with each other is that the missing or wrong information obtained in previous sections leads to misconceptions in the course of the proceedings. For this reason, it is necessary for the student to go to new subjects by eliminating the misconceptions of the past.

Keywords

Analytical Geometry, Concept , Error, Mathematics Education, Misconceptions

1. Introduction

Misconception is described as misunderstanding and means wrong or incomplete conception, any representation of a given concept in the mind but scientifically different from the definition of the concept (Hadjidemetriou and Williams, 2002). Consequently, misconception is defined as the body of behaviors stemming from students misbeliefs and experiences. Some errors might be symptomatic of a misconception, a prototypical way of thinking, or an intuition or mislearning and excessive load of information. Moreover, misconception may be beyond an error (Goris and Dyrenfurth 2009).

Students bring their alternative thoughts they have acquired before with them to the classroom (Fischbein, 1987; 1999). Since these concepts that are known by students have a certain integrity within them and are supported by some experiences of daily life, they are resistant to change and positive improvement (Simmons and Nelson; 2006). This has negative impacts on students learning other concepts related to the concept misunderstood by them (Kustos and Zelkowski, 2013).

As in all disciplines, instruction appropriate also for the structure of mathematics is directly related to students comprehending the mathematical concepts (Baykul, 2003). Previously learned concepts may present prerequisite of concepts to be learned later as mathematical subjects are interrelated (Hohmann, 1991). One of the reasons why students approach mathematics class with a prefixed idea is that mathematics involve abstractions. The main cause of this prefixed idea is the difficulties of perception in mathematics and use of the concepts (Ozkan, 2011). Perception of mathematical concepts incompletely or outside their scientific meanings is a universal problem to be addressed. One of the most important steps to be taken for brining several skills to the learners in mathematics education is to teach concepts properly (Breigheith and Kuncar 2002). If existing misconceptions are not eliminated and are transferred to next learning steps as they are, meaningful learning cannot occur. In this context, researchers address that misconceptions about certain concepts acquired by learners involuntarily or unwillingly should be eliminated and learners should confront the misconception (NRCS, 1997; Ryan and Williams, 2007). Tanner and Jonnes (2000) stated that existing misconceptions about a given subject need to be identified before teaching that subject and it should be identified where and why the learner has had the misconception.

There are several studies investigating the causes of misconceptions in mathematics learning and proposing solutions for eliminating the misconceptions. The following suggestions are made for eliminating misconceptions in these studies in short (Luchins and Luchins, 1985; Mason, 1989; Chang, 1995; Oberdorf and Taylor-Cox, 1999; Mikkilä-Erdmann, 2001; Cutugno and Spagnolo, 2002; Vlassis, 2004; Booth and Koedinger, 2008; Kazemi and Ghoraishi, 2012; Ozkan and Ozkan, 2012a; Ozkan and Ozkan, 2012b; Aygor and Ozdag, 2012; Lai and Wong, 2017).

- Concepts acquired in daily-life experiences and at school should not contradict each other.
- It should be ensured that students associated the concepts in their previously acquired knowledge with their new knowledge.
- Exemplification should be used frequently and examples should be chosen from those which students can face during their daily lives.
- Course books and teacher books that focus on concept education should be prepared.
- Misconceptions learned by students in the past should be absolutely corrected. New concepts should not be taught before correcting the old misconceptions.
- Teachers should guide students in every stage of concept instruction.
- How teachers take down notes of their experiences as feedbacks for possible misconceptions in future should guide novice teachers.
- Teacher opinions on misconceptions acquired by students in other subjects of mathematics can be received to determine the pedagogical causes.

It is also stated by researchers in the literature that learning diaries are effective in learners showing progress during the process and revealing their thoughts on the concepts (Hindman et al. 2004; Draper and Mclntosh, 2001; Ben-Hur, 2006; Harmin and Toth, 2006; Arter et al.; 2007).

This research identified students' misconceptions of analytic geometry which is a subject in mathematics curriculum of tenth-grade students in an effort. The study is composed of four sections. The next section addresses methodology of the study, the third section mentions the study results and the discussion, and finally, the final section involves the conclusion. tion of electronic products, and 3) conformity of style throughout a journal paper. Margins, column widths, line spacing, and type styles are built-in; examples of the type styles are provided throughout this document and are identified in italic type, within parentheses, following the example. Some components, such as multi-leveled equations, graphics, and tables are not prescribed, although the various table text styles are provided. The formatter will need to create these components, incorporating the applicable criteria that follow.

2. Method

2.1. Population

The research population was composed of tenth-grade students in İstanbul in the academic year of 2016-2017. The sample of the first part of the research was 2552 tenth-grade students who were studying at 19 most successful İstanbul high schools which apply the curriculum by Ministry of National Education. The sample of the second part was 299 students of two schools with the highest error rate within the 19 schools by their school averages and the last part used a sample of 10 students out of 299 students who made errors. Answers were chosen from previous years entering of national university exam.

2.2. Procedures

A three-stage procedure was followed to collect research data. In the first stage, 10 open-ended analytic geometry questions were asked so that the students could answer the questions in a more comfortable setting and the factor of chance could be reduced. In the second stage, the test version of the open-ended questions and finally an interview was performed.

The questions asked to the students covered the following subjects:

Question 1: Coordinate system: If the ordinate of E is 10, then find the apsis of D.

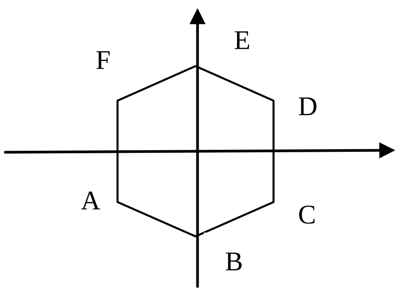

Figure 1. Question 1.

Question 2: Coordinate of a point: ABCD is the square. Find x + y.

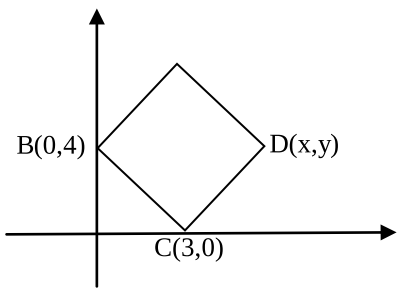

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Figure 2. Question 2.

Question 3: Relative position of two lines: l_1: 5x-2y+7=0 and l_2: 4x+my-3=0 are two lines. If $l_1 \perp l_2$, find m.

Question 4: Relative position of two lines: l_1: 3x-y=6 and l_2: 4x+(a+4)y=-6 are two lines. If $l_1 \; // \; l_2$, find a.

Question 5: Distance between two points: Find min($\sqrt{a^2+b^2}$).

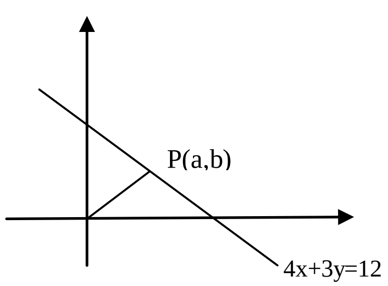

Figure 3. Question 5.

Question 6: Distance from a point to a line: Find the equation of the line of the median which starts from the vertex A of the triangle which has vertices O(0,0), A(8,0) and B(8,6).

Question 7: Finding the line given the equation: Find the equation of the line of the median which starts from the vertex A of the triangle which has vertices O(0,0), A(8,0) and B(8,6).

Question 8: Relative position of two lines: Find the equation of the line passing through the point of intersection of the lines l_1: $2x+3y-8=0$ and l_2: $7x+2y+16=0$ and the origin.

Question 9: Calculation of area between intersecting lines: If A(ABC) = 1 then find a.

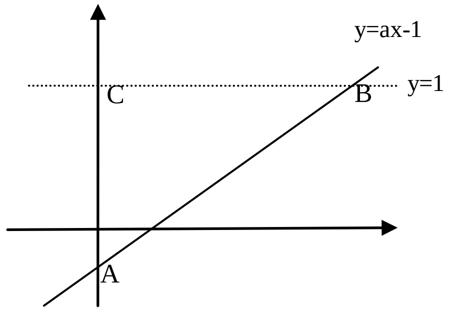

Figure 4. Question 9.

Question 10: If OA ⊥ AB and OB ⊥ AH then find the area of OAB

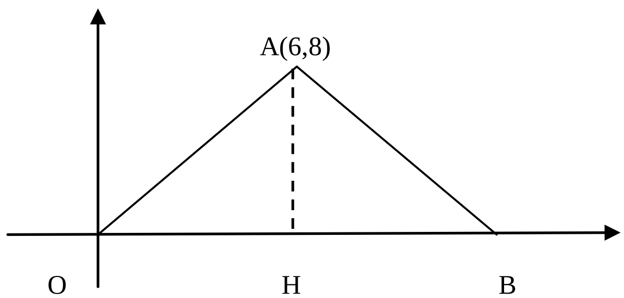

Figure 5. Question 10.

The Open-Ended Scale for Identifying Misconceptions was used in the first stage. The scoring applied for the open-ended questions are given in Table 1.

Table 1. Number of participant students in the sample schools.

Criteria	Score	Criteria	Score	Total Score
True answer	1	True Explanation	2	3
True answer	1	Almost True Explanation	1	2
True answer	1	Wrong Explanation	0	1
Wrong answer	0	True Explanation	2	2
Wrong answer	0	Wrong Explanation	0	0

The scores obtained in the test were used as the criterion of whether the students had any misconception. This criterion was determined in the form of triple grading. Accordingly, the score range of 0-2552 means a misconception, the score range of 2553-5104 means a partial misconception, and the score range of 5105-7656 means the lack of any misconception.

The number of correct, incorrect and non-answers were handled in the data analysis of the second stage. Number of correct answers within the range of 0-33.33% means "there is a misconception", 33.34%-66.67% means “there is a partial misconception”, and 66.68%-100% means “there is no misconception”.

10 interviews were evaluated in the final stage.

3. Test Results and Discussions

3.1. Open-Ended Questions

The results of the grading by whether the answers to the open-ended questions are correct, incorrect and by their justifications are presented in Table 2.

Table 2. Test scoring.

Questions	True Answer True Explanation	True Answer Almost True Explanation	True Answer Wrong Explanation	Wrong Answer True Explanation	Wrong Answer Wrong Explanation	Score
Question 1	874	15	0	43	348	2.738
Question 2	775	0	0	17	192	2.359
Question 3	1.714	26	0	283	130	5.760
Question 4	1.451	12	0	332	221	5.041
Question 5	491	13	0	473	569	2.445
Question 6	302	2	0	249	1.168	1.408
Question 7	784	3	0	217	769	2.792
Question 8	208	202	0	406	792	1.840
Question 9	225	67	146	477	1.124	1.909
Question 10	1.002	0	0	67	394	3.140
Total Score	7.826	340	146	2.564	5.707	29.432

According to Table 2, in a general overview of the questions, there were misconceptions in the five subjects of analytic geometry while partial misconceptions were observed in 4 questions and there was no misconception only in one question.

Whereas it was observed in the first question covering the coordinate system that there was a partial misconception, the scoring was close to the misconception. By whether the justifications are correct, about 36% of students' justifications are correct. It was observed that 30% of the students gave the correct answer to the Question 2 which covered the coordinate of a point while the rate of students who provided the correct justification was about 31%. It is seen that 85% of the students gave answer to the Question 2 about perpendicularity in the subject "relative position of two lines". 67% of those students gave the correct answer, and 78% of them had the correct justification. The scoring was about 75 and it can be clearly understood that there is no misconception of the subject. As for the answers to Question 4 about parallelism, about 70% of the students had the correct justification to their answers and there was a partial misconception while the scoring was closer to the upper limit.

The score of Question 5 (distance between two points) is in the range of 32% and this means that there was a misconception of the subject. Scoring of Question 6 which investigated whether there was a misconception of the subject "distance from a point to a line" remained in the part of 33%, which means there was a misconception of this subject. When examining the results of Question 7 covering the subject "finding the line given the equation", the scoring of about 70% shows that there was a partial misconception. It was found that there was a misconception among students in the subject "relative position of two lines" (Question 8). Whereas 62% of the students provided incorrect justification for the calculation of the area between intersecting lines (Question 9), 44% of the students provided both incorrect answer and justification. According to the scoring results, there is a clear misconception. In Question 10 covering the subject "finding the area of a triangle given corner points", 58% of the students did not answer, the scoring remained at 41.01%, meaning that there was a partial misconception.

3.2. Multiple Choice Questions

The multiple-choice questions asked to 299 students and the results are presented in Table 3.

Table 3. The Results of Multiple-Choice Questions.

Topic	Score	Score %	Misconception
Coordinate System	2,738	35.76	Almost
Coordinate of the mind	2,359	30.81	Yes
The Situation According to Two Directives (Parallelism and Secondary)	5,400	70.54	No
Distance Between Two Points	2,445	31.94	Yes
A Point is a Neutral	1,408	18.39	Yes
Finding Equivalent Equivalent	2,792	36.47	Almost
Situation of Two Directives According to One	1,840	24.03	Yes
Field Account between Intersecting Directives	1,909	24.93	Yes
Finding Corner Points of a Given Triangle	3,140	41.01	Almost

According to the results, there was no misconception of the subject "respective position of two lines (perpendicularity and parallelism) at the rate of 70.54%. On the other hand, there were partial misconceptions in the subjects of coordinate system (Question 1) at 35.76%, finding the line given the equation (Question 7) at 36.47% and finding the area of a triangle given corner points (Question 10) at 41.01%. It was seen that there were misconceptions in the remaining five subjects of coordinate of point (Question 2) at 30.81%, distance between two points (Question 5) at 31.94%, distance from a point to a line (Question 6) at 18.39%, relative position of two lines (Question 8) at 24.03%, and calculation of area between intersecting lines (Question 9) at 24.93%.

Test results show that there were no misconceptions of coordinate of point and relative position of two lines whatsoever. In addition, there were partial misconceptions in the subjects of distance between two points (Question 5), distance from a point to a line (Question 6), finding the line given the equation (Question 7), and calculation of area between intersecting lines (Question 9) while the rate of correct answers were below 50%. On the other hand, there were partial misconceptions in the subjects of coordinate system (Question 1), relative position of two lines (Question 8), and finding the area of a triangle given corner points (Question 9) whereas the rates were in upper limit.

3.3. Interview

It was seen in an interview with a student who answered the first question incorrectly that the student did not notice that the center of hexagon is in the origin. It was understood that the student who gave the incorrect answer to the second question saw the square in the shape of the question was situated to form an isosceles triangle with the origin and solved the question with the help of congruent triangle similarity. When it was reminded the student of that he/she could solve the third question by using the relation of condition of perpendicularity of two lines, it was observed that the student had not mastered the subject. In the interview with the student who gave the incorrect answer to the fifth question, it was seen that the student solve the question by predicting that the required point must be the midpoint of the abscissa values. It was observed that the student who solved the sixth question incorrectly had no knowledge of concept about how to find the shortest distance from a point to a line. The student who gave the incorrect answer to the seventh question also had insufficient knowledge of concept about the line equation. In the eighth question covering the relative position of two lines, the student added the line equations to each other. It was seen in the interview with the student who solve the final question incorrectly that the student misconceived the shape and answered the question by thinking that the shape was an isosceles triangle.

3.4. Discussions

It is seen in the first question that the misconception which the students had was that they thought the length of side must be equal to the abscissa value in the required point. When instructing this subject to the students, it was concluded that how the hexagon can be divided into congruent triangles with auxiliary lines and the congruent triangle relation should be reminded. In the misconception in the second question, it was seen that the students were deceived by the shape and perceived the shape in such a way that it was situated to form an isosceles triangle with the origin, therefore solving the question with the help of congruent triangle similarity. The students had lack of knowledge and contradiction in terms about the condition of perpendicularity in the third question. The misconceptions in the fifth question were about what linearity is and the conditions of linearity. As for the sixth question, the students had insufficient knowledge about the concept of the shortest distance from a point to a line and had misconceptions. It was seen that the misconception in the seventh question was about the subject of coordinates. The students had misconceptions about the line intersecting the given breakpoint and the origin given the equation of two lines. The students had insufficient knowledge of area in the ninth questions. The misconception in the tenth question was about the Euclidean relation. The following results were achieved at the end of the study.

- The errors in the test were higher than what had been anticipated. The students might not have taken the tests seriously.
- It was seen that the achievement rate was higher in the Science High Schools than in the Anatolian High School.
- While the rate of non-answers in the open-ended questions was very high, it was lower in the test questions. Likewise, the number of incorrect answers was lower in the tests while the number of correct answer was higher.
- Providing the students with auxiliary information in the tests increased the achievement. Hence, it became easier to observe misconceptions in the test questions.
- As the mathematical subjects are interrelated, incorrect or incomplete information acquired in previous subjects cause misconceptions in subjects later. Therefore, past misconceptions of students should be eliminated before passing towards new subjects.
- It cannot be ignored that one is not to be deceived by the shape or misleading information may be given about the shape in relevant questions.
- It was observed that the students had misconceptions of relative position of two lines. Hence, especially the concept of slope needs to be explained to students with different examples.
- In the subject of the shortest distance from a point to a line, the misconception was that the shortest distance would be achieved if the line was perpendicular. This showed that the students could not comprehend this concept, therefore having the misconception.
- A misconception of coordinate system was observed in the procedure. Such misconception can be mitigated if teachers

explain the concept of coordinate system with the help of shapes.

- Another misconception is the equation of line intersecting the point origin. The fact that there would be no constant in the equation of line intersecting the point origin should be highlighted in the subject of relative position of two lines. The student must learn the equation of line by drawing shapes.
- Negative number expressing a positive value as length in area calculations and equations of line is another subject in which misconceptions are observed.
- The concept of area calculation should be instructed using more visuals. Solutions can be shown by using different methods (matrix method, etc.).

4. Conclusions

In the first stage, an open-ended exam was applied to 2552 tenth-grade students studying at 19 high schools under İstanbul Provincial Directorate of National Education and 299 students from two high schools were tested in the second stage and 10 students were interviewed in the last stage in the academic year of 2016-2017. Errors and misconceptions of the students in the questions covering the analytic geometry were examined.

At the end, it was concluded that knowledge levels, errors and misconceptions of students in the analytic geometry should be identified to use proper instructional strategies. It is necessary to design different activities to improve the levels of students who cannot comprehend the analytic geometry on the level of their classrooms. This will ensure that the whole classroom achieves the same comprehension level. A decrease in errors and misconceptions will be observed and misconceptions will be identified more easily. Eliminating the misconceptions is possible by getting beyond the traditional instructional methods and keeping the teacher from the role of information transferer and the student from the role of passive listener.

Science of mathematics includes an order of definitions, axioms, theorems, and formulations. The mathematics education should follow this order. Definitions and theorems need to be learned properly without regarding mathematics as just a body of examples. Only then the students can learn the definition of the concept; otherwise, they stick only to the examples, memorize types of examples and the solution if they do not know the definition. This means memorization rather than learning. The way to avoid this is to teach mathematical theorems and concepts properly.

Each operation in mathematics involves the information of a concept previously acquired. When the operations are reinforced with conceptual knowledge, the student can explain not only how the question is solved but also why it is solved in a given way. Most of the errors indicate that the operations that are described as being mechanical have been learned but definitions and meanings of operations have not been comprehended. It was observed in the procedure that the students did random operations just to achieve a solution in the questions covering the subjects with poor foundation of conceptual knowledge. This can be prevented by students acquiring the concepts about the subject properly and teachers requesting explanation based on the knowledge of these concepts in each operation step when solving the question. Hence, students could learn to develop solutions according to the situation rather than memorizing the steps to be taken. This can be described as learning to think mathematically instead of memorizing the rules of mathematics

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