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Surfaces family with a common Mannheim asymptotic curve

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Abstract

In this paper, we analyzed surfaces family possessing a Mannheim partner of a given curve as a asymptotic. Using the Frenet frame of the curve in Euclidean 3-space, we express the family of surfaces as a linear combination of the components of this frame, and derive the necessary and sufficient conditions for coefficients to satisfy both the asymptotic and isoparametric requirements. The extension to ruled surfaces is also outlined. Finally, examples are given to show the family of surfaces with common Mannheim asymptotic curve.

Keywords

Asymptotic curve; Mannheim partner; Frenet Frame; Ruled Surface.

1. Introduction

At the corresponding points of associated curves, one of the Frenet vectors of a curve coincides with one of the Frenet vectors of other curve. This has attracted the attention of many mathematicians. One of the well-known curves is the Mannheim curve, where the principal normal line of a curve coincides with the binormal line of another curve at the corresponding points of these curves. The first study of Mannheim curves has been presented by Mannheim in 1878 and has a special position in the theory of curves (Blum, 1966). Other studies have been revealed, which introduce some characterized properties in the Euclidean and Minkowski space (Lee, 2011; Liu & Wang, 2008; Orbay &Kasap, 2009; Öztekin & Ergüt, 2011). Liu and Wang called these new curves as Mannheim partner curves: Let x and x_1 be two curves in the three dimensional Euclidean E^3. If there exists a corresponding relationship between the space curves x and x_1 such that, at the corresponding points of the curves, the principal normal lines of x coincides with the binormal lines of x_1, then x is called a Mannheim curve, and x_1 is called a Mannheim partner curve of x. The pair $\{x, x_1\}$ is said to be a Mannheim pair. They showed that the curve : $x_1(s_1)$ is the Mannheim partner curve of the curve $x(s)$ if and only if the curvature κ_1 and the torsion τ_1 of $x_1(s_1)$ satisfy following equation

$$\tau' = \frac{d\tau}{ds_1} = \frac{\kappa_1}{\lambda}\left(1+\lambda^2\tau_1^{\,2}\right)$$

for some non-zero constant λ .They also study the Mannheim curves in Minkowski 3-space. The generalizations of the Mannheim curves in the 4-dimensional spaces have been given (Matsuda & Yorozu, 2009; Akyiğit, *et al.* 2011). Later, Mannheim offset the ruled surfaces and dual Mannheim curves have been defined in Orbay *et al.*2009; Özkaldı *et al.* 2009; Güngör & Tosun, 2010). Apart from these, some properties of Mannheim curves have been analyzed according to different frames such as the weakened Mannheim curves, quaternionic Mannheim curves and quaternionic Mannheim curves of Aw(k) - type (Karacan, 2011; Okuyucu, 2013; Önder & Kızıltuğ, 2012; Kızıltuğ & Yaylı, 2015). In differential geometry, there are many important consequences and properties of curves (O'Neill, 1966; do Carmo, 1976). One of the most significant surface curves is an asymptotic curve. A curve on a surface is classified as asymptotic if its velocity always points in an asymptotic direction, that is, the direction in which the normal curvature is zero. Additionally, for a curve on a surface M to be asymptotic, its acceleration must always be tangent to M. In an asymptotic direction, the surface is not bending away from its tangent plane, at least instantaneously. On a surface, the Gaussian curvature is always non-positive along the asymptotic curve. Asymptotic curves on a surface have been a long-term research topic in differential geometry (Struik ,1961; O'Neill, 1966; Klingerberg, 1978). A useful interpretation of asymptotic directions and asymptotic curves is given by the Dupin indicatrix (Carmo, 1976). Hartman and Wintner (1951) showed that the degree of smoothness is an important notion in the classical theory of asymptotic curves of non-positive Gaussian curvature, which is usually left unspecified. Kitagawa (1988) proved that if M is a flat torus isometrically immersed in a unit 3-sphere, then all asymptotic curves on M are periodic. Garcia and Sotomayor (1996) studied the simplest qualitative properties of asymptotic curves of a surface immersed in Euclidean space. Garcia et al. 1996; studied immersions of surfaces into E^3 whose nets of asymptotic curves are topologically undisturbed under small perturbations of the immersion, which can then be described as structurally asymptotic stable. Asymptotic curves are also encountered in astronomy, astrophysics and architectural CAD. In order to find a set of escaping orbits of stars in a stellar system, it is necessary to find asymptotic curves of the Lyapunov orbits, because any small outward deviation from an asymptotic orbit will lead a star to escape from the system. Most previous work on asymptotic curves has focused on how to find such curves on a given surface. In practice, the more relevant problem is how to construct surfaces that contain a given spatial curve as a common asymptotic curve. In recent years, fundamental research has focused on the reverse problem, or backward analysis: given a 3D curve, how can we characterize those surfaces that possess this curve as a special curve, rather than finding and classifying curves on analytical curved surfaces. Wang et al. 2004; studied the problem of constructing a surface family from a given spatial geodesic. Lie et al. 2011; derived the necessary and sufficient condition for a given curve to be the line of curvature on a surface. Kasap et al. 2008; generalized the marching-scale functions of Wang and gave a sufficient condition for a given curve to be a geodesic on a surface. Bayram et.al. 2012; tackled the problem of constructing surfaces passing through a given asymptotic curve. Also, Bayram et.al. 2015; searched for a surface family possessing the natural lift of a given curve as a common asymptotic curve. They obtained the sufficient condition for the resulting surface to be a ruled surface and present a result for developable surfaces. Atalay and Kasap 2016; analyzed the problem of consructing a family ofsurfaces from a given some special Smarandache curves in using the Bishop frame and Frenet frame of the curve in Euclidean 3-space. Also they analyzed the problem of finding a surfaces family through an asymptotic curve with Cartan frame (Atalay and Kasap, 2015).

In this paper, we analyzed surfaces family possessing an Mannheim partner of a given curve as a asymptotic. Using the Frenet frame of the curve in Euclidean 3-space, we express the family of surfaces as a linear combination of the components of this frame, and derive the necessary and sufficient conditions for coefficents to satisfy both the asymptotic and isoparametric requirements. The extension to ruled surfaces is also outlined**.** Finally, examples are given to show the family of surfaces with common Mannheim asymptotic curve.

2. Preliminaries

Let E^3 be a 3-dimensional Euclidean space provided with the metric given by

$$\langle\, , \rangle = dx_1^{\,2} + dx_2^{\,2} + dx_3^{\,2}$$

where (x_1, x_2, x_3) is a rectangular coordinate system of E^3. Recall that, the norm of a arbitrary vector $X \in E^3$ is given by $\|X\| = \sqrt{\langle X, X\rangle}$. Let $\alpha = \alpha(s): I \subset IR \to E^3$ is an arbitrary curve of arc-length parameter s. The curve α is called a unit speed curve if velocity vector α' of a satisfies $\|\alpha'\| = 1$. Let $\{\mathrm{T(s), N(s), B(s)}\}$ be the moving Frenet frame along α, Frenet formulas is given by

$$\frac{d}{ds}\begin{pmatrix} T(s) \\ N(s) \\ B(s) \end{pmatrix} = \begin{pmatrix} 0 & \kappa(s) & 0 \\ -\kappa(s) & 0 & \tau(s) \\ 0 & -\tau(s) & 0 \end{pmatrix}\begin{pmatrix} T(s) \\ N(s) \\ B(s) \end{pmatrix},$$

where the function $\kappa(s)$ and $\tau(s)$ are called the curvature and torsion of the curve $\alpha(s)$, respectively.

Let C: $\alpha(s)$ be the Mannheim curve in E^3 parameterized by its arc length s and C*: $\alpha^*(s^*)$ is the Mannheim partner curve of C with an arc length parameter s^*. Denote by $\{T(s), N(s), B(s)\}$ the Frenet frame field along C: $\alpha(s)$, that is; $T(s)$ is the tangent vector field, $N(s)$ is the normal vector field, and $B(s)$ is the binormal vector field of the curve C respectively also denote by $\{T^*(s), N^*(s), B^*(s)\}$ the Frenet frame field along C^*: $\alpha^*(s)$, that is; $T^*(s)$ is the tangent vector field, $N^*(s)$ is the normal vector field, and $B^*(s)$ is the binormal vector field of the curve C^* respectively. If there exists one to one correspondence between the points of the space curves C and C^* such that the binormal vector of C is in the direction of the principal normal vector of the curve C^*, then the (C, C^*) curve couple is called Mannheim pairs (Liu & Wang, 2008).

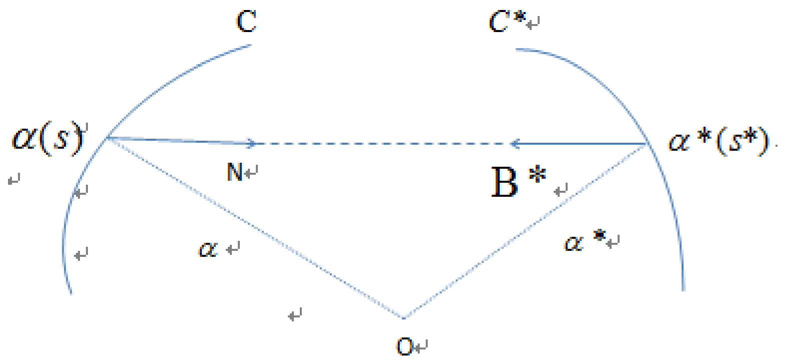

Figure 1. The Mannheim partner curves.

From the figure1, we can write $\alpha(s) = \alpha^*(s^*) + \lambda(s^*)B^*(s^*)$.

A curve α is an asymptotic curve on the surface $\varphi(s,v)$ if and only if along the curve the surface normal vector field $n(s,v_0)$ is orthogonal to the principal normal vector field N of the curve α. Equivalently, α is asymptotic if and only if $n(s,v_0)$ is parallel to the binormal vector field B. An isoparametric curve $\alpha(s)$ is a curve on a surface $\varphi(s,v)$ is that has a constant s or v-parameter value. In other words, there exist a parameter s_0 or v_0 such that $\alpha(s) = \varphi(s,v_0)$ or $\alpha(v) = \varphi(s_0,v)$. Given a parametric curve $\alpha(s)$, we call $\alpha(s)$ an isoasymptotic of a surface φ if it is both a asymptotic and an isoparametric curve on φ.

3. Surfaces with common Mannheim asymptotic curve

Suppose we are given a 3-dimensional parametric curve $\alpha(s)$, $L_1 \le s \le L_2$, in which s is the arc length and $\|\alpha''(s)\| \ne 0$. Let $\bar{\alpha}(s)$, $L_1 \le s \le L_2$, be the Mannheim partner of the given curve $\alpha(s)$.

Surface family that interpolates $\bar{\alpha}(s)$ as a common curve is given in the parametric form as

$$\varphi(s,v) = \bar{\alpha}(s) + [x(s,v)\bar{T}(s) + y(s,v)\bar{N}(s) + z(s,v)\bar{B}(s)],\ L_1 \le s \le L_2,\ T_1 \le v \le T_2, \tag{3.1}$$

where $x(s,v), y(s,v)$ and $z(s,v)$ are C^1 functions. The values of the marching-scale functions $x(s,v), y(s,v)$ and $z(s,v)$ indicate, respectively; the extension-like, flexion-like and retortion-like effects, by the point unit through the time v, starting from $\bar{\alpha}(s)$ and $\left\{\bar{T}(s), \bar{N}(s), \bar{B}(s)\right\}$ is the Frenet frame associated with the curve $\bar{\alpha}(s)$.

Our goal is to find the necessary and sufficient conditions for which the Mannheim partner curve of the unit space curve $\alpha(s)$ is an parametric curve and an asymptotic curve on the surface $\varphi(s,v)$.

Firstly, since Mannheim partner curve of the curve $\alpha(s)$ is an parametric curve on the surface $\varphi(s,v)$, there exists a parameter $v_0 \in [T_1, T_2]$ such that

$$x(s,v_0) = y(s,v_0) = z(s,v_0) = 0,\ L_1 \le s \le L_2,\ T_1 \le v_0 \le T_2. \tag{3.2}$$

Secondly, since Mannheim partner curve of $\alpha(s)$ is an asymptotic curve on the surface $\varphi(s,v)$, there exist a parameter $v_0 \in [T_1, T_2]$ such that

$$\left\langle \frac{\partial n(s,v_0)}{\partial s}, \overline{T}(s) \right\rangle = 0 \tag{3.3}$$

where n is a normal vector of $\varphi = \varphi(s,v)$ and $\overline{\mathrm{T}}$ is a normal vector of $\overline{\alpha}(s)$.

Theorem 3.1: Let $\alpha(s)$, $L_1 \le s \le L_2$, be a unit speed curve with nonvanishing curvature and $\overline{\alpha}(s)$, $L_1 \le s \le L_2$, be a Mannheim partner curve. $\overline{\alpha}$ is a asymptotic curve on the surface (3.1) if and only if

$$\begin{cases} \mathrm{x}(\mathrm{s},\mathrm{v}_0) = \mathrm{y}(\mathrm{s},\mathrm{v}_0) = \mathrm{z}(\mathrm{s},\mathrm{v}_0) = 0, \\ \dfrac{\partial \mathrm{z}(\mathrm{s},\mathrm{v}_0)}{\partial \mathrm{v}} = 0. \end{cases}$$

Proof : Let $\overline{\alpha}(s)$ be a Mannheim partner of the curve $\alpha(s)$. From (3.1) , $\varphi(s,v)$ parametric surface is defined by as follows:

$$\varphi(\mathrm{s},\mathrm{v}) = \overline{\alpha}(\mathrm{s}) + [\mathrm{x}(\mathrm{s},\mathrm{v})\overline{\mathrm{T}}(\mathrm{s}) + \mathrm{y}(\mathrm{s},\mathrm{v})\overline{\mathrm{N}}(\mathrm{s}) + \mathrm{z}(\mathrm{s},\mathrm{v})\overline{\mathrm{B}}(\mathrm{s})].$$

Let $\overline{\alpha}(s)$ Mannheim partner curve of the curve $\alpha(s)$ is an parametric curve on the surface $\varphi(s,v)$, there exists a parameter $v_0 \in [T_1, T_2]$ such that,

$$x(s,v_0) = y(s,v_0) = z(s,v_0) \equiv 0,\ L_1 \le s \le L_2,\ T_1 \le v_0 \le T_2\ (v_0\ fixed) \tag{3.4}$$

Secondly, since Mannheim partner curve of $\alpha(s)$ is an asymptotic curve on the surface $\varphi(s,v)$, there exist a parameter $v_0 \in [T_1, T_2]$ such that, from Eq. (3.3), we have

$$\left\langle \frac{\partial n(s,v_0)}{\partial s}, \overline{T}(s) \right\rangle = 0$$

The normal vector can be expressed as

$$\begin{aligned} n(s,v) = &\left[\frac{\partial z(s,v)}{\partial v}\left(\frac{\partial y(s,v)}{\partial s} + \overline{\kappa}(s)x(s,v) - \overline{\tau}(s)z(s,v) \right) - \frac{\partial y(s,v)}{\partial v}\left(\frac{\partial z(s,v)}{\partial s} + \overline{\tau}(s)y(s,v) \right) \right]\overline{T}(s) \\ &+ \left[\frac{\partial x(s,v)}{\partial v}\left(\frac{\partial z(s,v)}{\partial s} + \overline{\tau}(s)y(s,v) \right) - \frac{\partial z(s,v)}{\partial v}\left(1 + \frac{\partial x(s,v)}{\partial s} - \overline{\kappa}(s)y(s,v) \right) \right]\overline{N}(s) \\ &+ \left[\frac{\partial y(s,v)}{\partial v}\left(1 + \frac{\partial x(s,v)}{\partial s} - \overline{\kappa}(s)y(s,v) \right) - \frac{\partial x(s,v)}{\partial v}\left(\frac{\partial y(s,v)}{\partial s} + \overline{\kappa}(s)x(s,v) - \overline{\tau}(s)z(s,v) \right) \right]\overline{B}(s) \end{aligned} \tag{3.5}$$

Thus, if we let

$$\begin{cases}\phi_1(s,v_0)=\dfrac{\partial z(s,v_0)}{\partial v}\left(\dfrac{\partial y(s,v_0)}{\partial s}+\bar{\kappa}(s)x(s,v_0)-\bar{\tau}(s)z(s,v_0)\right)-\dfrac{\partial y(s,v_0)}{\partial v}\left(\dfrac{\partial z(s,v_0)}{\partial s}+\bar{\tau}(s)y(s,v_0)\right),\\ \phi_2(s,v_0)=\dfrac{\partial x(s,v_0)}{\partial v}\left(\dfrac{\partial z(s,v_0)}{\partial s}+\bar{\tau}(s)y(s,v_0)\right)-\dfrac{\partial z(s,v)}{\partial v}\left(1+\dfrac{\partial x(s,v_0)}{\partial s}-\bar{\kappa}(s)y(s,v_0)\right),\\ \phi_3(s,v_0)=\dfrac{\partial y(s,v_0)}{\partial v}\left(1+\dfrac{\partial x(s,v_0)}{\partial s}-\bar{\kappa}(s)y(s,v_0)\right)-\dfrac{\partial x(s,v_0)}{\partial v}\left(\dfrac{\partial y(s,v_0)}{\partial s}+\bar{\kappa}(s)x(s,v_0)-\bar{\tau}(s)z(s,v_0)\right).\end{cases}$$

We obtain

$$n(s,v_0)=\phi_1(s,v_0)\bar{T}(s)+\phi_2(s,v_0)\bar{N}(s)+\phi_3(s,v_0)\bar{B}(s) \tag{3.6}$$

We know that $\bar{\alpha}(s)$ is an asymptotic curve if and only if

$$\begin{aligned}&\left\langle \frac{\partial n(s,v_0)}{\partial s},\bar{T}(s)\right\rangle=0\\ &\Leftrightarrow \left\langle \frac{\partial}{\partial s}\left(\phi_1(s,v_0)\bar{T}(s)+\phi_2(s,v_0)\bar{N}(s)+\phi_3(s,v_0)\bar{B}(s)\right),\bar{T}(s)\right\rangle=0\\ &\Leftrightarrow -\bar{\kappa}(s)\phi_2(s,v_0)=0\end{aligned}$$

where, from (3.4), we have

$$\phi_2(s,v_0)=-\frac{\partial z(s,v_0)}{\partial v}$$

$$\Leftrightarrow \bar{\kappa}(s)\frac{\partial z(s,v_0)}{\partial v}=0$$

where $\bar{\kappa}$ is the curvature of the curve $\bar{\alpha}$. Since $\bar{\kappa}(s)\neq 0$, $L_1\leq s\leq L_2$,the curve $\bar{\alpha}$ is an asymptotic curve on the surface $\varphi(s,v)$ if and only if

$$\frac{\partial z(s,v_0)}{\partial v}=0 \tag{3.7}$$

Combining the conditions (3.4) and (3.7), we have found the necessary and sufficient conditions for the $\varphi(s,v)$ to have the Mannheim partner curve of the curve $\alpha(s)$ is an isoasymptotic.

Now let us consider other types of the marching-scale functions. In the Eqn. (3.1) marching-scale functions $x(u,v), y(u,v)$ and $z(u,v)$ can be choosen in two different forms:

1) If we choose

$$\begin{cases} x(s,v) = \sum_{k=1}^{p} a_{1k} l(s)^k x(v)^k, \\ y(s,v) = \sum_{k=1}^{p} a_{2k} m(s)^k y(v)^k, \\ z(s,v) = \sum_{k=1}^{p} a_{3k} n(s)^k z(v)^k, \end{cases}$$

then we can simply express the sufficient condition for which the curve $\overline{\alpha}(s)$ is an asymptotic curve on the surface $\varphi(s,v)$ as

$$\begin{cases} x(v_0) = y(v_0) = z(v_0) = 0, \\ a_{31} = 0 \ or \ n(s) = 0 \ \text{ or } \ \frac{dz(v_0)}{dv} = 0. \end{cases} \tag{3.8}$$

where $l(s), m(s), n(s), x(v), y(v)$ and $z(v)$ are C^1 functions, $a_{ij} \in IR$, $i = 1,2,3$, $j = 1,2,...,p$.

2) If we choose

$$\begin{cases} x(u,v) = f\left(\sum_{k=1}^{p} a_{1k} l(u)^k x(v)^k\right), \\ y(u,v) = g\left(\sum_{k=1}^{p} a_{2k} m(u)^k y(v)^k\right), \\ z(u,v) = h\left(\sum_{k=1}^{p} a_{3k} n(u)^k z(v)^k\right), \end{cases}$$

then we can write the sufficient condition for which the curve $\overline{\alpha}(s)$ is an asymptotic curve on the surface $\varphi(s,v)$ as

$$\begin{cases} x(v_0) = y(v_0) = z(v_0) = f(0) = g(0) = h(0) = 0, \\ a_{31} = 0 \ or \ n(s) = 0 \ \text{ or } \ \frac{dz(v_0)}{dv} = 0 \ or \ h'(0) = 0. \end{cases} \tag{3.9}$$

where $l(s)$, $m(s)$, $n(s)$, $x(v)$, $y(v)$, $z(v)$, f, g and h are C^1 functions.

Also conditions for different types of marching-scale functions can be obtained by using the Eqn. (3.4) and (3.7).

4. Ruled surfaces with common Mannheim asymptotic curve

Ruled surfaces are one of the simplest objects in geometric modelling as they are generated basically by moving a line in space. A surface φ is a called a ruled surface in Euclidean space, if it is a surface swept out by a straight line l moving alone a curve α. The generating line l and the curve α are called the rulings and the base curve of the surface, respectively.

We show how to derive the formulations of a ruled surfaces family such that the common Mannheim asymptotic is also the base curve of ruled surfaces.

Let $\varphi=\varphi(s,v)$ be a ruled surface with the Mannheim isoasymptotic base curve. From the definition of ruled surface, there is a vector $R=R(s)$ such that;

$$\varphi(s,v)-\varphi(s,v_0)=(v-v_0)R(s)$$

From (3.1), we get

$$(v-v_0)R(s)=x(s,v)\overline{T}(s)+y(s,v)\overline{N}(s)+z(s,v)\overline{B}(s)$$

For the solutions of three unknows $x(s,v)$, $y(s,v)$ and $z(s,v)$ we have,

$$x(s,v)=(v-v_0)\langle\overline{T}(s),R(s)\rangle$$

$$y(s,v)=(v-v_0)\langle\overline{N}(s),R(s)\rangle \qquad (4.1)$$

$$z(s,v)=(v-v_0)\langle\overline{B}(s),R(s)\rangle.$$

From (3.7) and (4.1), we have

$$\langle\overline{B}(s),R(s)\rangle=0 \qquad (4.2)$$

Including, $R(s)=x(s)\overline{T}(s)+y(s)\overline{N}(s)+z(s)\overline{B}(s)$ using (4.2) we obtain,

$$z(s)=0 \qquad (4.3)$$

So, the ruled surfaces family with common Mannheim isoasymptotic given by;

$$\varphi(s,v)=\overline{\alpha}(s)+v[x(s)\overline{T}(s)+y(s)\overline{N}(s)] \qquad (4.4)$$

5. Examples of generating simple surfaces with common Mannheim asymptotic curve

Example 5.1. Let $\alpha(s)=\left(3\cos\left(\frac{s}{5}\right),-3\sin\left(\frac{s}{5}\right),\frac{4}{5}s\right)$ be a unit speed curve. Then it is easy to show that

$$\begin{cases} T(s)=\left(-\frac{3}{5}\sin\left(\frac{s}{5}\right),-\frac{3}{5}\cos\left(\frac{s}{5}\right),\frac{4}{5}\right),\\ N(s)=\left(-\cos\left(\frac{s}{5}\right),\sin\left(\frac{s}{5}\right),0\right),\\ B(s)=\left(-\frac{4}{5}\sin\left(\frac{s}{5}\right),-\frac{4}{5}\cos\left(\frac{s}{5}\right),-\frac{3}{5}\right).\end{cases}$$

and the curvatures of this curve is $\kappa = \frac{3}{25}$ *and* $\tau = \frac{4}{25}$.

The parametric representation of the Mannheim partner (with respect to Frenet frame) curve $\overline{\alpha}(s)$ of the curve $\alpha(s)$ is obtained as (where $\lambda = \text{constant}$, for $\lambda = 1$)

$$\overline{\alpha}(s) = \alpha(s) - \lambda N(s)$$
$$= \left(4\cos\left(\frac{s}{5}\right), -4\sin\left(\frac{s}{5}\right), \frac{4}{5}s\right)$$

The Frenet vectors of the curve $\overline{\alpha}$ are found as

$$\begin{cases} \overline{T}(s) = \left(\frac{\sqrt{2}}{10}\sin\left(\frac{s}{5}\right), \frac{\sqrt{2}}{10}\cos\left(\frac{s}{5}\right), \frac{7\sqrt{2}}{10}\right), \\ \overline{N}(s) = \left(-\frac{7\sqrt{2}}{10}\sin\left(\frac{s}{5}\right), -\frac{7\sqrt{2}}{10}\cos\left(\frac{s}{5}\right), \frac{\sqrt{2}}{10}\right), \\ \overline{B}(s) = \left(-\cos\left(\frac{s}{5}\right), \sin\left(\frac{s}{5}\right), 0\right). \end{cases}$$

If we take $x(s,v) = y(s,v) \equiv 0,\ z(s,v) = \cos v - 1$ and $v_0 = 0$ then the Eqn. (3.4) and (3.7) are satisfied. Thus, we obtain a member of the surface with common Mannheim asymptotic curve $\overline{\alpha}(s)$ as

$$\overline{\varphi}(s,v) = \left(4\cos\left(\frac{s}{5}\right) - (\cos v - 1)\cos\left(\frac{s}{5}\right),\ -4\sin\left(\frac{s}{5}\right) + (\cos v - 1)\sin\left(\frac{s}{5}\right),\ \frac{4s}{5}\right)$$

Also, for $x(s,v) = y(s,v) \equiv 0,\ z(s,v) = \cos v - 1$ and $v_0 = 0$, we obtain a member of the surface with common asymptotic curve $\alpha(s)$ as

$$\varphi(s,v) = \left(3\cos\left(\frac{s}{5}\right) - \frac{4}{5}(\cos v - 1)\cos\left(\frac{s}{5}\right),\ -3\sin\left(\frac{s}{5}\right) - \frac{4}{5}(\cos v - 1)\cos\left(\frac{s}{5}\right),\ \frac{4s}{5} - \frac{3(\cos v - 1)}{5}\right)$$

where $-\pi \le s \le \pi,\ -1 \le v \le 1$ (Fig.2).

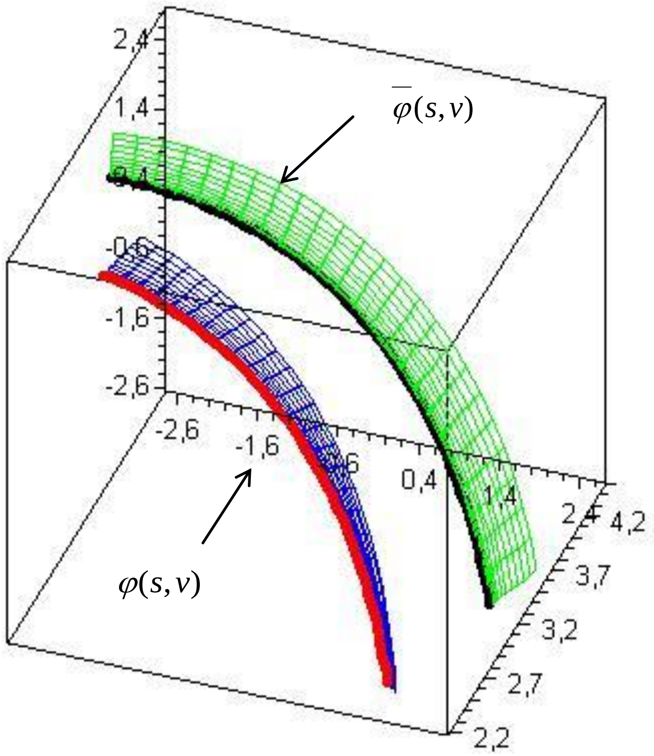

Figure 2. $\varphi(s,v)$ surface with curve $\alpha(s)$ and $\overline{\varphi}(s,v)$ Mannheim offset surface with Mannheim asymptotic curve ($\overline{\alpha}(s)$) of the curve $\alpha(s)$

If we take $x(s,v)=v\sin(s/5),\ y(s,v)=0,\ z(s,v)=v^2$ and $v_0=0$ then the Eqn. (3.4) and (3.7) are satisfied. Thus, we obtain a member of the surface with common Mannheim asymptotic curve as

$$\overline{\varphi}(s,v)=\begin{pmatrix} 4\cos\left(\frac{s}{5}\right)+\frac{\sqrt{2}}{10}v\sin\left(\frac{s}{5}\right)\sin\left(\frac{s}{5}\right)-v^2\cos\left(\frac{s}{5}\right),\ -4\sin\left(\frac{s}{5}\right)+\frac{\sqrt{2}}{10}v\sin\left(\frac{s}{5}\right)\cos\left(\frac{s}{5}\right) \\ +v^2\cos\left(\frac{s}{5}\right),\frac{4s}{5}+\frac{7\sqrt{2}}{10}v\sin\left(\frac{s}{5}\right) \end{pmatrix}$$

Also, for $x(s,v)=v\sin(s/5),\ y(s,v)=0,\ z(s,v)=v^2$ and $v_0=0$, we obtain a member of the surface with common asymptotic curve as

$$\varphi(s,v)=\begin{pmatrix} 3\cos\left(\frac{s}{5}\right)-\frac{4}{5}v\sin^2\left(\frac{s}{5}\right)-\frac{4}{5}v^2\sin\left(\frac{s}{5}\right),\ -3\sin\left(\frac{s}{5}\right)-\frac{3}{5}v\sin\left(\frac{s}{5}\right)\cos\left(\frac{s}{5}\right), \\ \frac{4s}{5}+\frac{4v}{5}\sin\left(\frac{s}{5}\right)-\frac{3}{5}v^2 \end{pmatrix}$$

where $-\pi\le s\le\pi,\ -1\le v\le 1$ (Fig. 3).

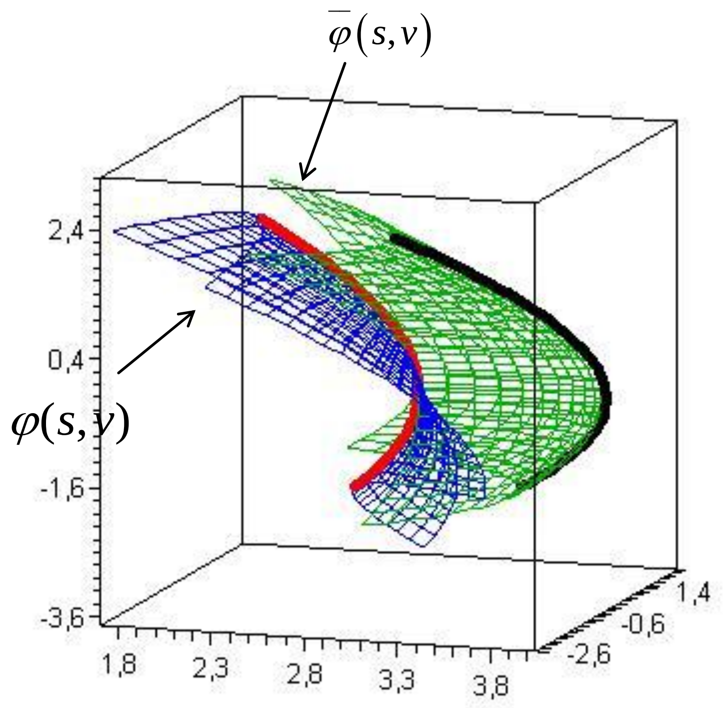

Figure 3. $\varphi(s,v)$ surface with curve $\alpha(s)$ and $\overline{\varphi}(s,v)$ Mannheim offset surface with Mannheim asymptotic curve ($\overline{\alpha}(s)$) of the curve $\alpha(s)$

If we take $x(s)=y(s)=s,\ z(s)=0$ and $v_0=0$ then the Eqn. (4.4) is satisfied. Thus, we obtain a member of the ruled surface with common Mannheim asymptotic curve as

$$\overline{\varphi}(s,v)=\left(4\cos\left(\frac{s}{5}\right)-\frac{3\sqrt{2}}{5}vs\sin\left(\frac{s}{5}\right),\ -4\sin\left(\frac{s}{5}\right)-\frac{3\sqrt{2}}{5}vs\cos\left(\frac{s}{5}\right),\ \frac{4s}{5}+\frac{4\sqrt{2}}{5}sv\right)$$

Also, for $x(s)=y(s)=s,\ z(s)=0$ and $v_0=0$,we obtain a member of the ruled surface with common asymptotic curve as

$$\varphi(s,v)=\left(3\cos\left(\frac{s}{5}\right)-\frac{3\sqrt{2}}{5}vs\sin\left(\frac{s}{5}\right),\ -3\sin\left(\frac{s}{5}\right)-\frac{3\sqrt{2}}{5}vs\cos\left(\frac{s}{5}\right),\ \frac{4s}{5}+\frac{4\sqrt{2}}{5}sv\right)$$

where $-\pi\le s\le\pi$, $-1\le v\le 1$ (Fig.4).

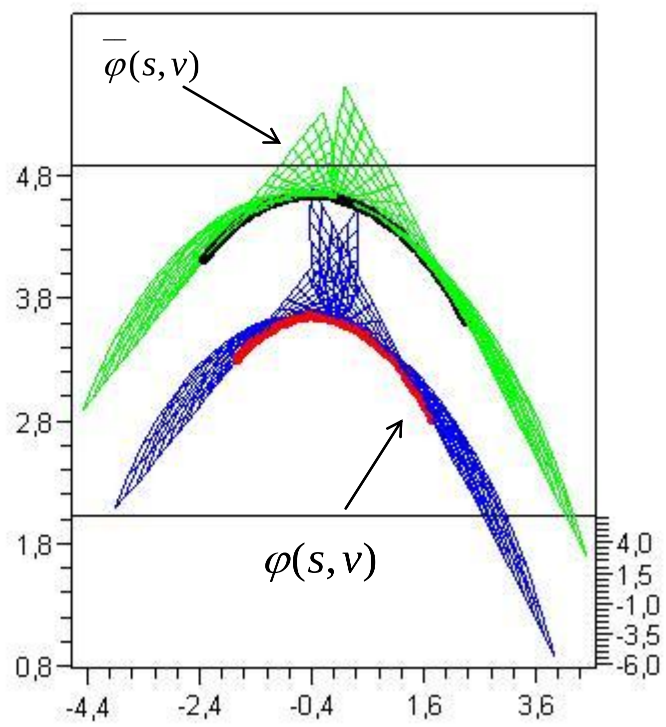

Figure 4. $\overline{\varphi}(s,v)$ as a member of the ruled surface and its Mannheim asymptotic curve, $\varphi(s,v)$ as a member of the ruled surface and its asymptotic curve

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